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Analytical Models for Battlespace Information Operations (BAT-10) Part 1

by

Donald P. Gaver
Patricia A. Jacobs

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Institute for Joint Warfare Analysis
NPS, Monterey, CA 93943

NAVAL POSTGRADUATE SCHOOL
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Donald P. Gaver and Patricia A. Jacobs

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Modern warfare uses information gathering resources ("sensors") and C4ISR capabilities to detect, acquire, and identify targets for attack ("shooters"). This report provides analytical state-space models that include the capabilities of the above functional elements in order to guide their appropriate balance; this includes attention to the effect of realistic errors, e.g. of target classification and battle damage assessment (BDA).

The great sensitivity of strike effectiveness to BDA error is described in the text and illustrated in Figures 3.12 – 3.15.

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Analytical Models for Battlespace Information Operations (BAT-IO)

Donald P. Gaver
Patricia A. Jacobs

Part 1

Some Nonlinear Dynamic Models of Strike Response to Region Defense

EXECUTIVE SUMMARY

The coordination of information acquisition and interpretation to direct force application is increasingly recognized as a crucial military systems design and investment issue. This paper illustrates tradeoffs between Blue/own regional Attacker sensor and shooter capabilities: it studies a deep strike <u>or</u> SCUD-hunting scenario in a low-resolution, aggregated manner using an analytical state-space approach that recognizes gross aggregated regional Defender (Red), and regional Attacker (Blue), system capabilities and limitations. Emphasis is accordingly placed on explicitly modeling the availability and utilization of information to a striking Attacker, as it becomes available from a realistically finite sensor and C2 capability. The (imperfect) information on opposition units, the Defenders, that are candidates for prosecution by the Attackers is passed to the finite, hence saturable (here missile-firing) Attacking force, the shooters, that then responds by prosecuting those units.

The models specifically recognize that regional Defenders will not be detected immediately, nor recognized perfectly, nor are Defender shots (e.g. SCUD launcher) fired perfectly, or immediately. Furthermore, attempts to effectively target are also realistically modeled as afflicted by *imperfect Attacker battle damage assessment* (BDA), an incapability that, if pronounced, will non-linearly saturate shooters, increase their response times, and hence reduce targeting effectiveness and efficient ammunition expenditures. Such models can allow for adaptation by both attackers and defenders to recent fortunes: if Defender presence and activity is effectively countered by Attackers, then the former may tend to be deterred or withdraw; if not, the Defenders are motivated to press their apparent advantage. Sharp, threshold-like, responses can follow from the possibly multi-stable dynamics. This behavior will be explored in a second report (Part II).

The present models are mainly deterministic or pseudo-stochastic in that they represent the non-linear *effect* of stochastic saturation approximately, but adequately. However, they can straightforwardly be "made stochastic", especially Markovian, and so realized using Monte Carlo simulations. Computer programs exist to provide numerical results; some are given. A simple one-dimensional stochastic (Markov birth-death) model is given as an appendix to Part II. This model can be shown analytically and numerically to exhibit "stochastic bi-stability" properties that under certain circumstances (parameter combinations) lead to *bimodal* steady-state distributions. Such a tendency will occur also in more detailed, but less analytically tractable models.

There are many problem elements that have been initially and purposefully ignored. They will be addressed in later work. For instance, the effects of different target types, false targets, and decoys must be added (some "decoys" are in effect present, in the form of killed Defenders, not so recognized, that are mistakenly re-targeted). The effect of different principles for Attacker target prioritization under uncertainty, i.e. dynamic scheduling, requires systematic attention. In the present models Attackers are invulnerable to attack; this is not always realistic, and can be changed to a duel-like scenario involving suppression of enemy (Defender) air defense (SEAD); a paper on this topic is in progress. In the current paper Attackers employ generic missiles only, but the use of (vulnerable, manned) Attack aircraft can similarly be modeled, as can combinations of Attack aircraft, Naval gunfire, and missiles, recognizing the coordination difficulties. Employment of cued reconnaissance aircraft, possibly UAVs, can likewise be represented quantitatively as state-space components. In addition, refinements that more faithfully represent spatial and perhaps other environmental constraints can be incorporated, as can details of communications assets and message-handling protocols in use by both Attackers and Defenders.

The present papers describe some of the possibilities for insights inherent in an enhanced state-space approach. As pointed out, many elaborations are possible. The objective is to recognize only that detail in the (preliminary) models that is sufficient to hint at payoff from adding suitable assets and strategies at appropriate points in the entire system. Finer detail and resolution is left to others to include, and possibly profit by. More elaborate and high-resolution models within such tools as NSS (METRON), and JWARS eventually can focus with greater intensity on some of the issues raised here.

In general we believe that this report is in accord with many of the views and suggestions of Ilachinski (1996), and also of Dockery and Woodcock (1993), and others. Those two publications contain many references, some to previous work on related topics.

Some Nonlinear Dynamic Models of Strike Response to Region Defense (BAT-IO)

Donald P. Gaver
Patricia A. Jacobs

1. Problem Formulation: Scenarios and Analytical Strategy

Consider this generic scenario: *Defenders* (Red units) enter a region, $\mathcal{R}$, in which they assemble, and that they wish to occupy and defend. Their purpose is to prepare to oppose the friendly/own (Blue *Attacker*) assets that, for example, may be intending to carry out an amphibious landing. Alternatively, the Red Defenders may simply intend to move about $\mathcal{R}$ and occasionally shoot harassing missiles in the fashion of TEL/SCUD systems at the time of the Gulf War; in this case the Blue Attacker facilities may be of lesser and different capability; see the OR/MS thesis of Munson (1996). Counter-fire could, in future, also at some stage come from a Navy force located near, but offshore from $\mathcal{R}$.

The region $\mathcal{R}$ is assumed to be under Blue surveillance; furthermore, each Red shot occasion is an opportunity for detection and response by a Blue Attacker. *Response* to Reds means that the Attacker shoots at the Defender after the latter has reached the "head of the line" of a queue or target list of others who have come to Attacker attention. Target priorities that order targets in urgency classes in the shooting queue are not yet considered here; the effect of such must be addressed subsequently. The priorities should be time and experience related, and established dynamically to recognize the uncertainty of target list identities. Also, all targets in the present model have the same value; this must be subject to change. The present model does account explicitly for possible shooter saturation or overload: the same is true for the overall Blue sensor force. And the analysis is purposefully made time-dependent, although simple steady-state results are of some interest to gain insight as to particular combinations of parameters.

Suppose further that Attacker (Blue) probability of (Red) kill is p_K; ideally, if a fired-upon Defender *is* killed, he/she is known to be no longer a threat; while, if missed, he/she returns to the pool of Defenders that are candidates for more "service", i.e. retargeting. Note that, more realistically, p_K might depend on the load or backlog of Defender-shooters revealed by Blue's sensors; the sensor, and Blue shooters' opportunities are enhanced by the Reds' recent shooting activities. The kill probability, p_K, could plausibly eventually decrease with increases in such load because of delays in prosecuting targets: such delays give opportunity for the target to move or evade. The initial model treats p_K as a *constant* (implicitly depending on only the average range of defender weapon fire that reaches the region of attacker location). This feature is a strong candidate for refinement in several ways.

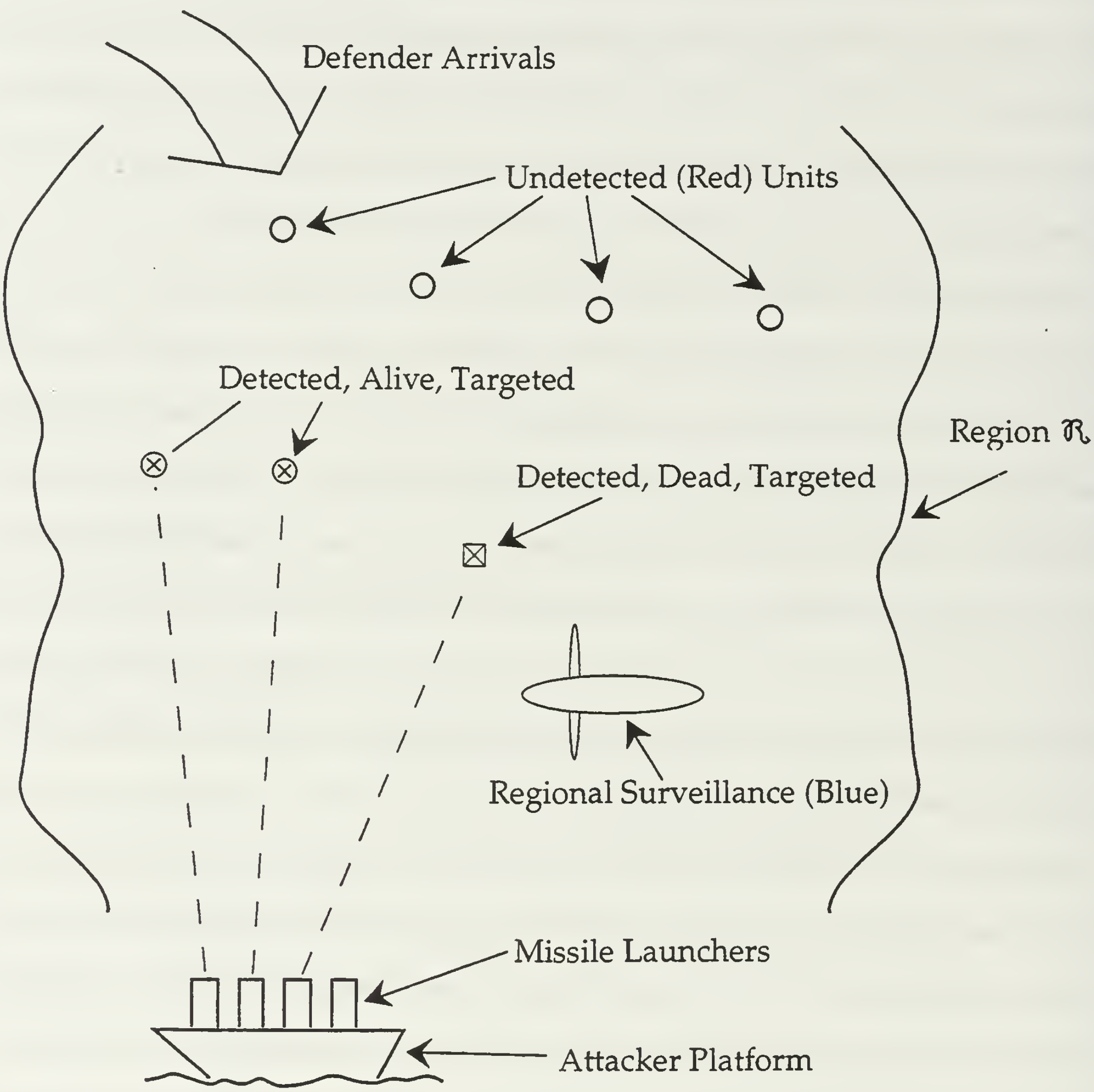
Our models specifically recognize that after a target is fired upon there is a *battle-damage assessment* (BDA) step taken by the Attacker force. Realistically, *BDA is imperfect*. In the present models BDA success is explicitly represented by conditional probabilities that reflect the chance of BDA error. One operating characteristic of BDA is the conditional probability that the Blue C4ISR system *concludes that* the target is *alive, given* that the Blue Attacker's response actually killed the Red target; denote this by c_{da}. Also there is a non-zero probability that the target is *perceived* by the Blue BDA system to be *dead*, given that the attack shot actually missed, so the target is actually *alive*; denote this error probability by c_{ad}. Note that if $c_{da} > 0$, as is realistic, an additional — and potentially unnecessary — load is gradually imposed on the Blue shooters, and this slows

Blue's effective response to Red presence and opposition potential. Of course, rapid invasion of the region by Defenders should give rise to many detections of Red units by Blue, which initially produces many opportunities but, if continued, subsequently loads up and may saturate the Blue Attack capability, reducing its responsiveness. Consequently it is important to *adapt* the Attacker force size and capabilities to adequately and cost-effectively control a perceived Defense intensity in the region. An adaptive control feature that regulates and allocates scarce attack assets has been added to our model sequence in Part II.

A further realistic feature of the present models is the possibility that Red units under surveillance may be lost, and hence returned to undetected status. Such a loss can be caused by a deliberate Red policy of occasionally leaving region $\mathcal{R}$ or hiding within it temporarily in order to shake off Blue pursuit and prosecution. Additionally, losses from surveillance can also occur inadvertently because of system inability to maintain effective contact, perhaps as a result of terrain properties, but also target action. The present models do not include the highly realistic presence of false targets, either of natural origin or deliberately introduced by Red as decoys. See Figure 1.1.

The purpose of this report is to provide first-step tools with which to investigate and describe certain tradeoffs and opportunities available in the sort of situation described. We begin by analyzing what may be the simplest reasonable analytical model(s) that suggest themselves. The models are here first exercised in a deterministic, approximately expected-value mode so as to quickly and efficiently explore for sensitivities or the "knees in the curves" beloved by analysts. In some cases quite simple but suggestive analytical (actually algebraic) solutions are possible; more comprehensive time-dependence effects are available by package computer programs that solve non-linear differential equations, e.g. MATLAB, or MATHEMATICA. Stochastic versions of some of the models have been formulated, and run in VISUAL BASIC; a simple example is available from the authors; it runs under Windows95. Such simulations promise

Region Defense and Attack



○ Undetected Red Units
⊗ Detected, Alive (possibly previously targeted)
⊠ Detected, Dead (BDA classified as alive; targeted)

Figure 1.1

to furnish preliminary rough but economical understanding of the rather complex, uncertain, and adaptive phenomena under study. The latter can guide further higher-resolution work, if desired. Computer run times are quite short in the models proposed, so exploration of alternatives is expedited.

The modeling strategy espoused is that of first exploring the conflict in an aggregated low-resolution broad-brush fashion using simple state-space models. This can be done quickly at minimal cost in the scarcest of analyst resources: time. Suggestions from such aggregated models can then be investigated at higher fidelity and at suitably detailed level using more elaborate models, such as perhaps NSS, GCAMS, or the NPS exploratory campaign model JWAEP, and, eventually, JWARS. We think of the present low-resolution models as tentative and subject to modification (often quickly and easily), both in response to the judgments and interests of analysts, and as a consequence of comparison with high-fidelity/resolution model runs; ideally, comparisons might even be made with exercise or true combat data. We believe that such interplay between model levels is a healthy and profitable way to carry out military modeling in support of analysis and understanding. Ultimately, models are formulated so as to enrich the vision and insights of analysts, at best providing insights at relatively low cost in time and money.

2. Model I: Surveillance Target Classification (Delayed Saturable BDA)

Consider this model: Defending Red units enter region $\mathcal{R}$ at a specified rate. They are the subject of a prescribed level of surveillance/reconnaissance, are identified (possibly incorrectly) as alive, and are eventually targeted by members of a group of Attacking Blue shooters. As a result, some are killed and remain in the region unclassified until revisited by the surveillance system; at this point the error-prone classification service occurs, and a dead Red may be classified as alive and is ultimately (and wastefully) retargeted. Live Red units in the region are similarly found by the sensor system and targeted, perhaps after several previous attempts have been made to shoot them. If a Defender is a TEL, and if it fires a (SCUD) missile, it is presumed to be quickly detected (although its track may be lost before prosecution). When an unclassified but dead Red unit in the region is finally classified as dead that unit is in future considered to be non-existent; this may take considerable time, during which it is effectively a decoy.

Red units may be lost to Blue's surveillance because of a variety of Red actions: for example, a TEL may take refuge in a prepared hideout, perhaps under a bridge. Such losses to detection and surveillance are assumed in the models to occur at a (user-specified) constant rate. Units, once lost, are subject to reacquisition at the same rate as newly arrived units. Units in hiding presumably cannot fire weapons (launch SCUDs).

The setup describes a certain kind of (imperfect) delayed information processing concerning the perceived state of both newly-arrived and previously engaged live targets, but also the state of targeted Defenders that *have been killed but not yet classified.*

The above setup represents a system that includes a form of effectively delayed BDA; it is one that depends on the general rate of surveillance of the region of interest. A subsequent model addresses the implications of a classification system tied more closely to the shooters.

Parameters of Model I

$\lambda(t)$ = Defender arrival rate into region $\mathfrak{R}$. The assumption of a *constant* arrival rate actually permits an explicit steady-state solution to be found, for what that is worth. One can perhaps string such together to represent various stages of arrival, during which the system reaches quasi-stationarity. The model also represents a situation in which an initial number of red Defenders is present in $\mathfrak{R}$, with no reinforcements.

μ = Service rate of an attacker ($1/\mu$ = mean time for an Attacker to track, shoot, flight time of missile).

p_K = Probability an attacked target is killed. This parameter is presently taken to be range-dependent only on average.

v = Track-loss rate ($1/v$ = mean of a holding time of Defender in track). Once a unit track is lost, it is no longer a viable target for Blue service.

α = Defender (e.g. TEL = SCUD launcher) shoot rate ($1/\alpha$ = mean time between shots by a single Defender). This parameter is irrelevant under certain circumstances.

γ = Rate Defenders leave region ($1/\gamma$= mean holding time of Defender unit in region), or hide within region.

ξ = Rate undetected/tracked Defenders are acquired by sensors/C2 system.

s_D = Number of Attacker-shooters; these are viewed as individual servers that engage Defender targets for a given time.

r_{aa} = Probability that a target that is alive is classified as being alive by the sensors/C2 system. $1 - r_{aa} = r_{ad} = P(\text{target classified or perceived as dead} \mid \text{alive})$.

r_{da} = Probability that a dead target is misclassified as being alive.

Any or all of the above parameters can be dependent upon time, at the discretion of the analyst. We maintain constants for very preliminary steady-state analysis.

State Variables for Model I

The following state variables are needed to describe the present dynamical system:

$R_u(t)$ = Number of *undetected* live (hence potentially active and threatening) Red Defenders present in region $\mathfrak{R}$ at time t.

$R_d(t)$ = Number of *detected* live Red Defenders present at time t. These are on the Blue Attacker-shooter's target list, and will be engaged unless lost by the sensor system (they may go into hiding, or even leave the region covered by the surveillance, e.g. JSTARS).

$D(t)$ = Number of *detected and perceived to be alive,* hence potentially Blue-Attacker-targeted, but actually *dead* Red Defenders at t. These are present because Blue battle damage assessment (BDA) is realistically imperfect.

$M(t)$ = Number of dead Defenders in the region that are not yet classified. Classification is done by the Blue surveillance/reconnaissance system.

For the deterministic modeling of saturable service by the Attack shooters (presumably missiles in the present context, although Attack aircraft can also be represented) we make use of the following approximation, cf. Filipiak (1988), Agnew (1976), and also Rider (1967), for the rate of processing by the Blue shooters,

$$H_S(t) \equiv H\left(R_d(t) + D(t); \mu, s_D\right)$$

$$= \frac{\left[R_d(t) + D(t)\right]\mu s_D}{1 + \left[R_d(t) + D(t)\right]}.$$

State Transition Equations for Model I

$$\frac{dR_u(t)}{dt} = \underbrace{\lambda(t)}_{\substack{\text{Arrival}\\ \text{rate}}} + \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t)[1-p_K]}_{\substack{\text{Blue service rate of live Defenders}\\ \text{that results in failure/missed shot}}} + \underbrace{\nu R_d(t)}_{\substack{\text{Rate of}\\ \text{track loss}}}$$

$$- \underbrace{R_u(t)(\alpha+\gamma)}_{\substack{\text{Defenders leaving region}\\ \text{and detections caused by}\\ \text{Red activity } (\alpha)}} - \underbrace{\xi r_{aa} R_u(t)}_{\substack{\text{Surveillance/recon.}\\ \text{system detection and}\\ \text{correct classification}\\ \text{(as alive)}}} \qquad (2.1a)$$

$$\frac{dR_d(t)}{dt} = \underbrace{\xi r_{aa} R_u(t)}_{\substack{\text{Surv./Recon.}\\ \text{detection and}\\ \text{classification}\\ \text{as alive}}} + \underbrace{\alpha R_u(t)}_{\substack{\text{Detection}\\ \text{by Red}\\ \text{action}}} - \underbrace{(\gamma+\nu) R_d(t)}_{\substack{\text{Rate of leaving}\\ \text{region and track}\\ \text{loss}}} - \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Rate of Blue shooting}\\ \text{at live Defenders}}} \qquad (2.1b)$$

$$\frac{dD(t)}{dt} = \underbrace{\xi r_{da} M(t)}_{\substack{\text{Rate of Surv./Recon.}\\ \text{detection and}\\ \text{incorrect classification}\\ \text{of dead targets}}} - \underbrace{\frac{D(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Rate of shooter service}\\ \text{of dead (misclassif.}\\ \text{as live) Defenders}}} \qquad (2.1c)$$

$$\frac{dM(t)}{dt} = -\xi\Big(\underbrace{r_{dd}}_{\substack{\text{Dead}\\ \text{classif.}\\ \text{correctly}\\ \text{(permanently)}}} + \underbrace{r_{da}}_{\substack{\text{Dead}\\ \text{classif.}\\ \text{as}\\ \text{live}}} \Big) M(t)$$

$$+ \underbrace{\frac{D(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Service of dead}\\ \text{but perceived}\\ \text{as live Defenders}}} + \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t) p_K}_{\substack{\text{Service rate of live}\\ \text{Defenders that results}\\ \text{in success/kill}}} \qquad (2.1d)$$

Long-Run Solutions

In order to investigate the possible long-run or steady-state behavior of the various state variables, and to compute answers to some interesting operational questions, look at solutions to the equations obtained by setting the derivatives equal to zero when $\lambda(t) = \lambda$, a constant; these are fixed points to which, under certain conditions, the system will eventually settle.

$$0 = \lambda + \frac{R_d}{R_d + D} H_S[1 - p_K] + vR_d - R_u(\alpha + \gamma) - \xi r_{aa} R_u \tag{2.2a}$$

$$0 = \xi r_{aa} R_u + \alpha R_u - (\gamma + v)R_d - \frac{R_d}{R_d + D} H_S \tag{2.2b}$$

$$0 = \xi r_{da} M - \frac{D}{R_d + D} H_S \tag{2.2c}$$

$$0 = -\xi M + \frac{D}{R_d + D} H_S + \frac{R_d}{R_d + D} H_S p_K \tag{2.2d}$$

Solving for M in (2.2c) and substituting the result into (2.2d) results in

$$D = R_d \frac{r_{da}}{r_{dd}} p_K \equiv c R_d. \tag{2.3}$$

Multiplying (2.2b) by $(1 - p_K)$ and adding to (2.2a) results in

$$0 = \lambda + \left(vp_K - \gamma(1 - p_K)\right)R_d - R_u\left(\alpha p_K + \gamma + \xi r_{aa} p_K\right). \tag{2.4}$$

Thus,

$$R_u = K_1 + K_2 R_d \tag{2.5}$$

where

$$K_1 = \frac{\lambda}{\alpha p_K + \xi r_{aa} p_K + \gamma} \tag{2.6a}$$

$$K_2 = \frac{vp_K - \gamma(1 - p_K)}{\alpha p_K + \xi r_{aa} p_K + \gamma}. \tag{2.6b}$$

Special Case: $\gamma = 0$ (rate at which alive Defenders leave the region is 0)

If $\gamma = 0$, then adding equations (2.2a) – (2.2b) results in

$$0 = \lambda - \frac{R_d}{R_d + D} H_S p_K \tag{2.7}$$

$$0 = \lambda - \frac{1}{1+c}\mu s_D \frac{(1+c)R_d}{1+(1+c)R_d} p_K \tag{2.8}$$

where c is defined in (2.3).

Thus,

$$0 = \lambda + \lambda(1+c)R_d - \mu s_D R_d p_K. \tag{2.9}$$

Hence, the condition for the system to be stable is

$$\frac{\lambda}{\mu s_D p_K}\left(1 + p_K \frac{(1-r_{dd})}{r_{dd}}\right) < 1. \tag{2.10}$$

The steady-state number of active Defenders in the region when $\gamma = 0$ and (2.10) holds is

$$\begin{aligned} R_u + R_d &= K_1 + (K_2+1)R_d \\ &= \frac{\lambda}{\alpha p_K + \xi r_{aa} p_K} + \frac{((\alpha+v)+\xi r_{aa})}{\alpha + \xi r_{aa}} \frac{\frac{\lambda}{\mu s_D p_K}}{1 - \frac{\lambda}{\mu s_D p_K}\left[1 + p_K \frac{r_{da}}{1-r_{da}}\right]}. \end{aligned} \tag{2.11}$$

Discussion

Examination of the final (right-most) term in (2.11) shows that, if the BDA error probability r_{da} is at all substantial, the queue of waiting targets (many actually dead) skyrockets non-linearly. This is the result of wasteful retargeting. It is therefore a prime technical objective of the Attacker to reduce r_{da}; otherwise it will be necessary for Attacker/Blue to shoot faster (or with greater effect), wastefully using up its ammunition inventory. The same general effect can also occur if the Defender region departure or concealment rate, γ, is positive but relatively small compared to Defender's action rate, α. If this is not so the current model would permit Defenders to, somewhat futilely, enter the region and leave without shooting. This latter tactic actually might be feasible if only to escape possible detection, or, if suspected to be detected, to escape before actual engagement. *Or*, simply to tie up Attacker forces that might otherwise be used elsewhere.

General Case: $\gamma > 0$

If $\gamma > 0$, then R_d satisfies the quadratic equation

$$0 = (\lambda - \gamma K_1) + R_d\left[(\lambda - \gamma K_1)(1+c) - \mu s_D p_K - \gamma(K_2+1)\right] - \gamma(K_2+1)(1+c)R_d^2. \quad (2.12)$$

Define

$$\begin{aligned} q &= \left[(\lambda - \gamma K_1)(1+c) - \mu s_D p_K - \gamma(K_2+1)\right]^2 + 4(\lambda - \gamma K_1)\gamma(K_2+1)(1+c) \\ &= (\lambda - \gamma K_1)^2 \left\{ \left[(1+c) - \left[\frac{\mu s_D p_K + \gamma(K_2+1)}{\lambda - \gamma K_1} \right] \right]^2 + \frac{4(K_2+1)(1+c)}{\lambda - \gamma K_1} \right\}. \end{aligned} \quad (2.13)$$

It can be shown that

$$\lambda - \gamma K_1 = \frac{\lambda(\alpha + \xi r_{aa})p_K}{(\alpha + \xi r_{aa})p_K + \gamma} \quad (2.14)$$

and that the quadratic equation for R_d, (2.12), will have only one positive root, and this provides the long-run value of R_d. The R_u comes from (2.5), and that of D from (2.3).

3. Model II: Simplified Surveillance with Immediate BDA

We introduce next a model that parsimoniously represents salient features of the posited long-range Blue response to the Red Defense of $\mathcal{R}$ assembly situation. It is also simple enough to permit explicit algebraic solution for long-run or steady-state behavior, if such exists. The latter formulas allow very convenient automated exploration by such devices as ANTS (Active Nonlinear Tests of Complex Simulation Models), developed by J.H. Miller of Carnegie-Mellon University and the Sante Fe Institute.

In this model there is assumed to be additional ability for the shooter server to conduct BDA immediately after firing.

Additional Parameters of Model II

c_{aa} = Probability that a Defender that is alive is classified as alive immediately after it has been fired upon. $1 - c_{aa} = c_{ad} = P$(Defender classified or perceived as dead | alive).

c_{da} = Probability that a dead Defender is misclassified as being alive immediately after being fired upon. $1 - c_{da} = c_{dd} = P$(Defender classified or perceived as dead | dead).

The state variables considered are these:

State Variables for Model II

We need the following variables to describe system evolution:

$R_u(t)$ = Number of *undetected* live (hence potentially active and threatening) Red Defenders present in region $\mathcal{R}$ at time t.

$R_d(t)$ = Number of *detected* live Red Defenders present at time t. These are on the Blue Attacker-shooter's target list, and will be engaged unless lost by the sensor system (some may go into hiding, or even leave the region covered by Blue surveillance, e.g. JSTARS).

$D(t)$ = Number of *detected and perceived to be alive,* hence potentially Blue-targeted, but actually *dead* Red Defenders at t. These are present because Blue battle damage assessment (BDA) is realistically imperfect.

In this model Blue Attackers ignore the Red Defenders that are dead and once-perceived to be dead; if a dead target is ever so classified, it is thereafter omitted from consideration. In this model, classification (BDA) is modeled as carried out soon (immediately) after a Blue Attacker engages/shoots at a Red Defender.

State Transition Equations for Model II

The following are transition equations for evolution of the system state.

$$\begin{aligned}\frac{dR_u(t)}{dt} = & \underbrace{\lambda(t)}_{\substack{\text{Arrivals}\\ \text{into region}}} \\ & + \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H\big(R_d(t)+D(t);\mu,s_D\big)}_{\text{Blue service / shoot rate (at live Red target)}} \cdot \underbrace{(1-p_K)}_{\substack{\text{Miss}\\ \text{Probability}}} \cdot \underbrace{c_{ad}}_{\substack{\text{Mis-class}\\ \text{("dead" back}\\ \text{to undetected)}}} \\ & + \underbrace{R_d(t)\nu}_{\substack{\text{Alive lost by}\\ \text{surveillance}\\ \text{system}}} - \underbrace{R_u(t)\xi r_{aa}}_{\substack{\text{New}\\ \text{detections}\\ \text{(sensors)}}} - \underbrace{R_u(t)\alpha}_{\substack{\text{New detections}\\ \text{caused by}\\ \text{Red activity}\\ \text{(shooting)}}} - \underbrace{R_u(t)\gamma}_{\substack{\text{Red Defenders}\\ \text{leave region}}}\end{aligned} \qquad (3.2a)$$

$$\begin{aligned}\frac{dR_d(t)}{dt} = & \underbrace{R_u(t)\xi r_{aa}}_{\substack{\text{New sensor}\\ \text{detections}}} + \underbrace{R_u(t)\alpha}_{\substack{\text{Activity}\\ \text{detections}}} - \underbrace{R_d(t)\gamma}_{\substack{\text{Leave}\\ \text{region}}} - \underbrace{R_d(t)\nu}_{\substack{\textit{Alive}\text{ lost by}\\ \text{surveillance}\\ \text{system}}} \\ & - \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H\big(R_d(t)+D(t);\mu,s_D\big)}_{\text{Serv./shoot rate}} \cdot \Big[\underbrace{(1-p_K)c_{ad}}_{\substack{\text{Miss \&}\\ \text{misclass}}} + \underbrace{p_K}_{\text{Kill}}\Big]\end{aligned} \qquad (3.2b)$$

$$\frac{dD(t)}{dt} = \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H(R_d(t)+D(t);\mu,s_D)\cdot[p_K c_{da}]}_{\substack{\text{Blue kills an alive Red,}\\ \text{misclass. as alive}}} - \underbrace{\frac{D(t)}{R_d(t)+D(t)} H(R_d(t)+D(t);\mu,s_D)\cdot c_{dd}}_{\substack{\text{"Re-kill" dead classed alive;}\\ \text{class as dead}}} \tag{3.2c}$$

where

$$H\left(R_d(t)+D(t);\mu,s_D\right) = \frac{\left[R_d(t)+D(t)\right]\mu s_D}{1+\left[R_d(t)+D(t)\right]}.$$

Solutions

In order to investigate the long-run behavior of the various state variables, and to compute answers to some interesting operational questions, look at solutions to the equations obtained by setting the derivatives equal to zero when $\lambda(t) = \lambda$, a constant; these are the fixed points to which, under certain conditions, the system will eventually settle.

$$0 = \lambda + \frac{R_d}{R_d+D}[q_K c_{ad}]H(R_d+D;\mu,s_D) + vR_d - (\alpha+\gamma)R_u - \xi r_{aa}R_u \tag{3.3a}$$

$$0 = \alpha R_u + \xi r_{aa}R_u - (\gamma+v)R_d - \frac{R_d}{R_d+D}(q_K c_{ad} + p_K)H(R_d+D;\mu,s_D) \tag{3.3b}$$

$$0 = \frac{R_d}{R_d+D}[p_K c_{da}]H(R_d+D;\mu,s_D) - \frac{D}{R_d+D}c_{dd}H(R_d+D;\mu,s_D) \tag{3.3c}$$

where

$$q_K = 1 - p_K.$$

If the third equation (3.3c) is solved first we find

$$D = \frac{p_K(1-c_{dd})R_d}{c_{dd}} \tag{3.4}$$

so that

$$\frac{R_d}{R_d + D} = \frac{1}{1 + \frac{p_K(1-c_{dd})}{c_{dd}}}. \tag{3.5}$$

To eliminate the nonlinear term from the first two equations (3.3a) – (3.3b) multiply the first by $(q_K c_{ad} + p_K)$ and the second by $q_K c_{ad}$ and add to obtain

$$0 = \left[\lambda + vR_d - (\xi r_{aa} + \alpha + \gamma)R_u\right](q_K c_{ad} + p_K) + \left[(\xi r_{aa} + \alpha)R_u - (\gamma + v)R_d\right]q_K c_{ad}. \tag{3.6}$$

Simplify:

$$0 = \lambda(q_K c_{ad} + p_K) + (vp_K - \gamma q_K c_{ad})R_d - \left[\gamma q_K c_{ad} + (\xi r_{aa} + \alpha + \gamma)p_K\right]R_u. \tag{3.7}$$

Thus,

$$R_u = \frac{\lambda(q_K c_{ad} + p_K)}{\gamma q_K c_{ad} + (\xi r_{aa} + \alpha + \gamma)p_K} + \frac{(vp_K - \gamma q_K c_{ad})}{\gamma q_K c_{ad} + (\xi r_{aa} + \alpha + \gamma)p_K} R_d \tag{3.8}$$

$$\equiv L + MR_d. \tag{3.9}$$

To find R_d, insert (3.9) and (3.4) into (3.3b) and solve for R_d.

Special Case: $\gamma = 0$ (the rate at which alive Defenders leave the region = 0).

Let

$$c = \frac{p_K(1-c_{dd})}{c_{dd}} \tag{3.10}$$

Assume $\gamma = 0$.

Adding equations (3.3a) – (3.3b) results in

$$0 = \lambda + \frac{1}{1+c} q_K c_{ad} \mu s_D \frac{(1+c)R_d}{1+(1+c)R_d} - \frac{1}{1+c}(q_K c_{ad} + p_K)\mu s_D \frac{(1+c)R_d}{1+(1+c)R_d}$$

$$= \lambda - \mu s_D p_K \frac{R_d}{1+(1+c)R_d} \tag{3.11}$$

$$= \lambda\left[1+(1+c)R_d\right] - \mu s_D p_K R_d$$

$$= \lambda + R_d\left\{\lambda(1+c) - \mu s_D p_K\right\}.$$

Hence,

$$0 = \lambda + \frac{1}{1+c} q_K c_{ad} \mu s_D \frac{(1+c)R_d}{1+(1+c)R_d} - \frac{1}{1+c}(q_K c_{ad} + p_K)\mu s_D \frac{(1+c)R_d}{1+(1+c)R_d}$$

$$= \lambda - \mu s_D p_K \frac{R_d}{1+(1+c)R_d} \tag{3.12}$$

$$= \lambda[1+(1+c)R_d] - \mu s_D p_K R_d$$

$$= \lambda + R_d\{\lambda(1+c) - \mu s_D p_K\}.$$

Thus, for the system to be stable

$$\frac{\lambda}{\mu s_D p_K}\left[1 + p_K \frac{(1-c_{dd})}{c_{dd}}\right] < 1. \tag{3.13}$$

In the unrealistic but instructive case in which Defenders never leave the region we can thus see directly the effects of BDA misclassification. The steady-state number of active Defenders in the region when $\gamma = 0$ is

$$R_u + R_d = L + (M+1)R_d$$

$$= \frac{\lambda(q_K c_{ad} + p_K)}{(\xi r_{aa} + \alpha)p_K} + \frac{(v + \xi r_{aa} + \alpha)}{(\xi r_{aa} + \alpha)} \frac{\dfrac{\lambda}{\mu s_D p_K}}{1 - \dfrac{\lambda}{\mu s_D p_K}\left[1 + p_K \dfrac{c_{da}}{1-c_{da}}\right]} \tag{3.14}$$

We see from the last term in (3.14) that increases in c_{da} can result in more sizable, rapid, and non-linear increases in the total active Defenders, $R_u + R_d$, than do increases in c_{ad}; the latter is also influential but only linearly. This behavior occurs because $c_{da} > 0$ results in unnecessary work by the shooting server and can result in system overload. The C2 server is recognized to be equivalent to a deterministic infinite server, which cannot be saturated — a feature that is a candidate for change. This model differs only slightly from Model I but it is simpler.

Comparison of (3.14) with (2.11) in the case in which $r_{da} = c_{da}$ shows that the alive Red population, $R_u + R_d$, is smaller for the present immediate BDA model, Model II. In this case R_u is smaller for Model II than Model I, while R_d is the same for both models.

General Case: $\gamma > 0$

If $\gamma > 0$, then R_d satisfies the quadratic equation

$$0 = (\lambda - \gamma L) + [(\lambda - \gamma L)(1+c) - \mu s_D p_K - \gamma(M+1)]R_d - \gamma(M+1)(1+c)R_d^2. \quad (3.15)$$

Define

$$\begin{aligned} q &= [(\lambda - \gamma L)(1+c) - \mu s_D p_K - \gamma(M+1)]^2 + 4(\lambda - \gamma L)\gamma(M+1)(1+c) \\ &= (\lambda - \gamma L)^2 \left\{ \left[(1+c) - \left[\frac{\mu s_D p_K + \gamma(M+1)}{(\lambda - \gamma L)} \right] \right]^2 + 4\frac{(M+1)(1+c)}{\lambda - \gamma L} \right\}. \end{aligned} \quad (3.16)$$

It can be shown that

$$\lambda - \gamma L = \frac{\lambda(\xi r_{aa} + \alpha)p_K}{\gamma q_K c_{ad} + (\xi r_{aa} + \alpha + \gamma)p_K} > 0, \quad (3.17)$$

and that the quadratic equation for R_d (3.15) will have one positive root. That positive root is given by

$$R_d = \frac{(\lambda - \gamma L)(1+c) - \mu s_D p_K - \gamma(M+1) + \sqrt{q}}{2\gamma(M+1)(1+c)} \quad (3.19)$$

Note that c is given by (3.10), M by (3.8), q by (3.16). From this formula we can get an explicit algebraic expression for R_u from (3.8), and for D from (3.4).

Numerical Examples

Two measures of evaluation (MOE) are the rate of Defender attrition

$$\rho_A = \frac{R_d}{R_d + D} \frac{R_d + D}{1 + R_d + D} \mu s_D p_K = \frac{R_d}{1 + R_d + D} \mu s_D p_K$$

and the rate of Defender firing (shooting)

$$\varphi = (R_u + R_d)\,\alpha.$$

Figures 3.1 – 3.11 present results of the two models. Figures 3.1 – 3.10 plot rate of Defender firing, φ, versus rate of Defender attrition, ρ_R, for various values of α, the firing rate per Defender. The value of each rate is plotted with a symbol for each value of α. The model parameters for Figures 3.1 – 3.6 are $\lambda = 10$, $p_K = 0.7$, $\gamma = 0.5$, $v = 20$, $r_{da} = r_{ad} = c_{da} = c_{ad} = 0.3$. In each figure the rates of firing per Defender, α, are 0.1, 0.5, 1.0, 1.5, ..., 4.5, 5. In Figures 3.1 – 3.3, the rate of Attacker-shooter service, $\mu_{SD} = 25$. In Figures 3.4 – 3.6, the rate of Attacker-shooter service, $\mu_{SD} = 50$. In Figures 3.1 and 3.4, the Attacker-sensor acquisition rate $\xi = 1$. In Figures 3.2 and 3.5, the Attacker-sensor acquisition rate $\xi = 5$. In Figures 3.3 and 3.6, the sensor acquisition rate $\xi = 10$.

The immediate BDA model always results in a higher rate of Defender attrition for the same rate of Defender firing. However, the difference becomes negligible for large ξ, the rate at which sensors acquire the Defenders. The difference is larger when the rate of shooting service, $\mu_{SD} = 50$, (in Figures 3.4 – 3.6) than when $\mu_{SD} = 25$, (in Figures 3.1 – 3.3) for the same values of ξ. This behavior results from the increased number of undetected Defenders, R_u, in the delayed BDA model. Note that for small ξ, the increase in the rate of Defender attrition is more responsive to change in the firing rate per Defender, α; Defenders are then more often discovered when they reveal themselves. For large surveillance rate, ξ, the rate of Defender attrition is almost constant as a function of α. Not surprisingly, for small ξ, a Defender is more likely to be detected just after it fires than before.

In Figure 3.7, $v = 0$, $\xi = 1$, $\mu_{SD} = 10$ and $\mu_{SD} = 25$ with the other parameters the same. Note that when $\mu_{SD} = 10$, the difference between immediate BDA and delayed BDA is negligible. This is because the Attacking-shooting server is saturated although the acquisition rate ξ is relatively small. The effect of increasing the missile firing rate to $\mu s_D = 25$ is to decrease the rate of Defender shooting by decreasing R_d; further, since the shooting server is not now saturated there is a larger difference in the rate of Defender attrition between immediate and delayed BDA.

In Figure 3.8, $\xi = 1$, $\mu s_D = 25$ and ν, the rate at which detected Defenders are lost from track, is set equal first to 0 and then to 10. Note the anticipated higher rate of Defender attrition for $\nu = 0$, and also the accompanying decrease in rate of Defender shooting attributable to the decrease in $R_u + R_d$, the Defender population available to fire missiles.

In Figure 3.9, $\nu = 0$, $\mu s_D = 10$ and ξ, the rate of surveillance/reconnaissance, is first set equal to 1, and then raised to 20. There is no apparent difference between immediate and delayed BDA for $\xi = 20$; live targets are reacquired relatively quickly. Further, as α increases, the saturation of the shooting server makes the rate of Defender attrition equal for $\xi = 1$ and $\xi = 20$. The present Blue Attacking shooter force can not profit by the increased acquisition capability.

In Figure 3.10, $\xi = 10$, $\nu = 20$, $\lambda = 10$, $\gamma = 0.5$, $p_K = 0.7$ and $\mu s_D = 25$. One curve, with larger rate of Defender attrition, corresponds to $r_{ad} = r_{da} = c_{ad} = c_{da} = 0.3$. The lower curve corresponds to higher error probabilities, arbitrarily $r_{ad} = r_{da} = c_{ad} = c_{da} = 0.5$. The less-effective BDA results in a decrease of about 5 in the rate of Defender attrition and an increase of about 10 in the rate of Defender shooting rate for $\alpha = 4.0$, 4.5 and 5. The difference between immediate and delayed BDA is small because of the comparatively high rate of Blue surveillance/reconnaissance, $\xi = 10$.

In Figure 3.11, $\lambda = 10$, $\xi = 1$, $p_K = 0.7$, $\nu = 20$ and $us_D = 25$. The rate at which Defenders leave the region is $\gamma = \theta\alpha$ where $\alpha = 1$ and $\theta = 0.1, 0.2, 0.5, 1, 2, \ldots, 10$. If $\theta < 1$ then delayed BDA results in a larger Defender shooting rate than immediate BDA. As the rate at which Defenders leave the region becomes larger than the firing rate per Blue Attacker, the rates of Defender shooting for immediate BDA and delayed BDA become essentially the same. The rate of Defender attrition is always below the rate of Defender shooting but the rates become comparable as the rate of Defender departure from the region becomes greater.

Numerical Example: Nonstationary Results

In this example there is a maximum number of Red Defenders, M, which enter the region at a linearly increasing rate, $\lambda(t) = \lambda t$, for $t \le T_M$ where $T_M = \sqrt{\frac{2M}{\lambda}}$; note that the total number of arrivals is $\int_0^{T_M} \lambda s\, ds = M$.

In Figures 3.12 – 3.15, the total number of Defenders $M = 50$ and $\lambda = 5$; thus, $T_M = \sqrt{20}$ is the time at which all Defenders have entered the region. Other parameters are as follows: the track loss rate $\nu = 0.5$; the detection rate $\xi = 0.1$; the Red activity rate $\alpha = 0.5$; the Blue service rate $\mu s_D = 12$; the probability of kill $p_K = 0.7$; the probability a live Red target detected by a Blue sensor is classified as live, $r_{aa} = 0.8$; and the probability Blue classifies an alive Red as dead, $c_{ad} = 0.7$. The Defenders do not leave the area; $\gamma = 0$.

In Figures 3.12 – 3.13, the probability with which Blue correctly classifies a dead Red as dead is $c_{dd} = 0.2$. This rather low figure is reflected in a dramatic growth in the backlog of already-dead targets and in a correspondingly drawn out campaign. In Figures 3.14 – 3.15, $c_{dd} = 0.8$. Figures 3.12 and 3.14 present the number of alive Red and dead Red targets (but not yet classified as dead) waiting for or being served by the Blue shooters for Model II. When $c_{dd} = 0.2$, the number of dead Red targets that are misclassified as alive and are awaiting retargeting is much larger than the number of live targets waiting to be served by the Blue shooters. The Blue shooters are saturated by the misclassified dead Red targets. Figures 3.13 and 3.15 present the accumulated number of Blue shots, Red shots, and Reds that are killed for Model II; also presented is the number of Red Defenders alive at time t. Comparison of Figures 3.13 and 3.15 indicates that the saturation of the Blue shooters by dead Red targets when $c_{dd} = 0.2$ decreases Blue's ability to prosecute alive Red targets. This impairment results in Red Defenders being alive for a far longer period of time and being able to shoot many more times. Further, Blue wastes much ammunition retargeting dead Red targets.

4. Model III: Command and Control *Delays* Explicitly Represented

The previous models are next extended to reflect realistic *delays* in classification and communication of defender detection to shooters. Although delays were modeled before, the very realistic effects of non-linear congestion and queuing was not explicitly represented. It may be seen that additional state variables are now required to minimally specify the dynamical state of the system. As before,

$R_u(t)$ = Number of *undetected* live (potentially active) Defenders present in the region at time t,

$R_a(t)$ = Number of newly-acquired live Defenders present at time t,

$R_d(t)$ = Number of *detected* live/functional Defenders present at time t,

$D(t)$ = Number of *detected and misclassified as live*, but actually dead, Defenders present at time t.

The Defenders enumerated by $R_d(t)$ and $D(t)$ are viewed as targetable, i.e. eligible for Attacker engagement; they are effectively "queued up" for shooter service. Additionally, we wish to define and enumerate those Defenders that have been engaged and await classification as to damage status (BDA) and possible attacker response or service:

$S_a(t)$ = Number of live *unclassified* Defenders that have been engaged/shot at by Attackers, at t,

$S_d(t)$ = Number of dead *unclassified* Defenders present at time t.

Note that all such variables might be treated as state variables of a multidimensional birth-death Markov stochastic process. A mathematically explicit treatment of such a setup appears cumbersome, but simulation models have been written to allow the effects of randomness to be investigated.

Parameters of the Model

T = Maximum number of Defenders allowed in the region simultaneously by the Defender decision makers. This parameter is, in effect, a control variable.

λ = Arrival rate of Defender (targets) to area. Note that the overall arrival rate is here allowed to depend on a hypothetical goal for the Defenders, T. Additions are made on the basis of current Defender count deficiency from the goal level.

μ = Rate at which Defenders are "served" by an Attacker-shooter.

s_D = Number of shooters; these are viewed as individual servers that engage Defender targets for a given time.

η = Rate at which Defenders are served by the Attacker's C2 system, viewed as a (saturable) service subsystem.

s_U = Number of C2 servers possessed by the Attacker force.

ν = Individual rate at which Defenders (Red targets) are lost from track but are still in the region.

α = Individual rate at which Defenders are active (e.g. shooting), hence causing potential damage but also revealing their presence; equivalently, $1/\alpha$ is the expected time between shots by a Defender.

ξ = Rate of acquisition of Defenders by Attacker surveillance (a sweep-width concept).

γ = Rate at which Defenders leave the region; equivalently $1/\gamma$ is the expected time that a Red Defender spends in the region (if it is not killed).

p_K = Single-shot probability with which a Blue Attacker kills a Red Defender.

c_{aa} = Probability that an engaged Defender that is still alive is correctly classified as being alive; $c_{ad} = 1 - c_{aa}$ is the probability of misclassification of a live Defending target as dead.

c_{da} = Probability that an engaged Defender that has been killed (is dead) is misclassified as being alive; $c_{dd} = 1 - c_{da}$ is the probability of correctly classifying a dead Defending target as dead.

Saturable C2 Service Submodels

For the deterministic modeling of saturable service by the C2 facility and by the Attacker-shooters (presumably missile launchers in the present context) we make use of the following approximations, (again Filipiak (1988)): for the C2 rate of service, i.e. of refining and processing information concerning those Defenders detected, we use

$$\begin{aligned} H_C(t) &\equiv H\big(R_a(t)+S_a(t)+S_d(t);\eta,s_C\big) \\ &= \frac{\big[R_a(t)+S_a(t)+S_d(t)\big]\eta s_C}{1+\big[R_a(t)+S_a(t)+S_d(t)\big]} \end{aligned} \tag{4.1}$$

and for the rate of processing by shooters,

$$\begin{aligned} H_S(t) &\equiv H\big(R_d(t)+D(t);\mu,s_D\big) \\ &= \frac{\big[R_d(t)+D(t)\big]\mu s_D}{1+\big[R_d(t)+D(t)\big]}. \end{aligned} \tag{4.2}$$

Both of these expressions reflect the saturability of the respective service systems. It is argued, e.g. for the C2 service system, that the load or backlog on the system is the sum of (a) $R_a(t)$, the number of acquired Defenders never before engaged; (b) $S_a(t)$, these previously-engaged and *alive* but unclassified at t; and (c) $S_d(t)$, these Defenders previously engaged that are *dead* but unclassified at t. These constitute the entire load seen by the C2 system; that load is processed at *individual* rate η, so for small load the gross processing rate is proportional to that load, while if that load grows the gross service rate saturates at the overall gross rate ηs_C, as it should. An identical argument holds for the shooter service facility behavior.

We also incorporate a static (time and event independent) weighting/control scheme that prioritizes the server attention to the different classes of targets. Again for the C2 service, we allow different C2 service emphasis on, for instance, the newly-detected Defender targets that have not yet been engaged than on the previously engaged, but not-yet-damage-classified. The weights can be made to adapt to changing backlogs and opportunities, i.e. to represent "emerging behavior", but no details are given at present.

State Transition Equations

Here are the state transition equations proposed to represent the enhanced model.

$$\frac{dR_u(t)}{dt} = \underbrace{\lambda\left[1-\frac{R_u(t)+R_a(t)+R_d(t)+S_a(t)}{T}\right]}_{\substack{\text{Defender entry rate to region}\\ \text{(or emergence from hiding).}\\ \text{Controlled by Red}}} + \underbrace{\frac{w_S S_a(t)+w_A R_a(t)}{w_a R_a(t)+w_S\left(S_a(t)+S_d(t)\right)}}_{\substack{\text{Rate that unclassified live targets}\\ \text{are designated by C2 as dead}\\ \text{(back to undiscovered status)}}} H_C(t)\cdot c_{ad}$$

$$+ \underbrace{\nu\left(R_a(t)+R_d(t)+S_a(t)\right)}_{\substack{\text{Rate of loss of track of}\\ \text{Defenders under track at } t}} - \underbrace{(\gamma+\alpha+\xi)R_u(t)}_{\substack{\text{Rate of loss: leave region } (\gamma)\text{;}\\ \text{detection as a consequence of}\\ \text{Defender action } (\alpha)\text{; detection of}\\ \text{Defenders from surveillance action } (\xi)}} \tag{4.3}$$

$$\frac{dR_a(t)}{dt} = \underbrace{(\alpha+\xi)R_u(t)}_{\substack{\text{Detection of Red}\\ \text{Defenders by their action } (\alpha)\\ \text{and by Blue surveillance } (\xi)}} - \underbrace{(\gamma+\nu)R_a(t)}_{\substack{\text{Rate of leaving}\\ \text{region } (\gamma) \text{ or track loss}\\ \text{or temporary hiding } (\nu)}}$$

$$- \underbrace{\frac{w_A R_a(t)}{w_A R_a(t)+w_S\left(S_a(t)+S_d(t)\right)} H_C(t)}_{\text{Rate of C2 system classification}} \tag{4.4}$$

$$\frac{dR_d(t)}{dt} = \underbrace{\frac{w_A R_a(t)+w_S S_a(t)}{w_A R_a(t)+w_S\left(S_a(t)+S_d(t)\right)} H_C(t) c_{aa}}_{\text{Rate of C2 system classification of live targets as alive}}$$

$$- \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Rate of engagement of}\\ \text{live, classified Defenders}}} - \underbrace{(\nu+\gamma)R_d(t)}_{\substack{\text{Rate at which track}\\ \text{lost } (\nu) \text{ or region left } (\gamma)\\ \text{by live classified Defenders}}} \tag{4.5}$$

$$\frac{dS_a(t)}{dt} = \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t)(1-p_K)}_{\substack{\text{Rate at which eligible live Defender targets}\\ \text{are engaged and missed, becoming}\\ \text{candidates for classification}}} - \underbrace{\frac{w_S S_a(t)}{w_A R_a(t) + w_S\left(S_a(t)+S_d(t)\right)} H_C(t)}_{\substack{\text{Rate of correct classification of}\\ \text{live unclassified Defenders}}} - \underbrace{(\nu+\gamma) S_a(t)}_{\substack{\text{Rate of track loss } (\nu)\\ \text{or leaving region } (r)}} \tag{4.6}$$

$$\frac{dS_d(t)}{dt} = \underbrace{\frac{R_d(t)}{R_d(t)+D(t)} H_S(t) p_K}_{\substack{\text{Rate at which eligible live}\\ \text{Defenders are engaged and killed}}} - \underbrace{\frac{w_S S_d(t)}{w_A R_a(t) + w_S\left(S_a(t)+S_d(t)\right)} H_C(t)}_{\substack{\text{Rate at which unclassified dead Defenders}\\ \text{are (correctly) classified as dead}}} + \underbrace{\frac{D(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Rate at which dead, but}\\ \text{misclassified as alive, Defenders}\\ \text{are engaged, rejoining the C2}\\ \text{classification queue}}} \tag{4.7}$$

$$\frac{dD(t)}{dt} = \underbrace{\frac{w_S S_d(t)}{w_A R_a(t) + w_S\left(S_a(t)+S_d(t)\right)} H_C(t) c_{da}}_{\substack{\text{Rate at which Defenders that are dead but}\\ \text{misclassifed as alive are serviced by C2 and,}\\ \text{again, misclassified as alive}}} - \underbrace{\frac{D(t)}{R_d(t)+D(t)} H_S(t)}_{\substack{\text{Rate of engagement of}\\ \text{Defenders that are dead,}\\ \text{but misclassified as alive}}} \tag{4.8}$$

Numerical Examples

Table 1 presents steady-state results of Model III with parameters $T = 100$, $\lambda = 10$, $p_K = 0.7$, $\nu = 0.5$, $\gamma = 0.5$, $\alpha = 1$, $c_{ad} = c_{da} = 0.3$, $w_S = w_A = 1$. Displayed are rates of Defender shooting, ρ_S, and rates of Defender attrition ρ_A where

$$\rho_S = \alpha\left(R_u + R_a + R_d + S_a\right)$$

$$\rho_A = \mu s_D \frac{R_d}{1+\left(R_d + D\right)}.$$

Note that increasing the acquisition rate ξ from 1 to 10 when $\eta s_U = 10$ and $\mu s_D = 10$ or 20 does not change the Defender shooting and attrition rates by much; this behavior is due to saturation of the C2 server. Increasing that capacity, as measured by ηs_U, to 30 allows the Defender shooting and attrition rates to become more sensitive to changes in ξ and μs_D.

TABLE 1
Defender Shooting Rates and Defender Attrition Rates
for Model with C2 Server
$T = 100, \lambda = 10, p_K = 0.7, \nu = 0.5, \gamma = 0.5, \alpha = 1, c_{ad} = c_{da} = 0.3, w_S = w_A = 1$

ξ	1				10			
μs_D	10 Defender Rate of		20 Defender Rate of		10 Defender Rate of		20 Defender Rate of	
	Shooting	Attrition	Shooting	Attrition	Shooting	Attrition	Shooting	Attrition
ηs_U								
10	12.7	2.4	12.4	2.6	12.6	2.5	12.3	2.6
20	10.3	3.8	9.1	4.6	9.8	4.1	8.4	5.0
30	9.7	4.2	7.6	5.5	8.9	4.7	5.6	6.6

5. Concluding Discussion

The equations describing the models in this report have been solved numerically, beginning with various hypothetical initial conditions, and the results are intuitively agreeable. Various functionals of these solutions that are operationally meaningful and informative can also be numerically evaluated; e.g. the (expected) number of weapons fired by each side up to tie t; this is the important MOE *inventory expenditure*. Cumulative attrition can also be computed, as well as the effect of attacks SCUD firings. It is the dependence of such MOEs upon system properties, such as surveillance capability, including the adequacy of BDA assessment, and attrition capability by defense shooters that the models will help to clarify. Tradeoffs can be uncovered, and the operational value of certain proposed system enhancements revealed. We plan to cover this exploration phase more extensively in later work.

In Part II of this report series we study the effect of *adaptive* policies that regulate both attacker and defender changes in region $\Re$ occupancy as a consequence of experience. Some such control phenomenon is dependent on perception, which may well be faulty. In subsequent work still more realistic issues are included.

Finally, it is emphasized that this modeling effort is purposefully aggregated and broad-brush so that quick turnaround results are possible. The deterministic models presented here can be very quickly exercised on modern PCs. The simulation models that follow these are similarly quickly run. Such a feature means that considerable exploration is possible before the high-resolution digging begins. We are convinced that the preliminary spadework described can efficiently locate paydirt for later exploitation.

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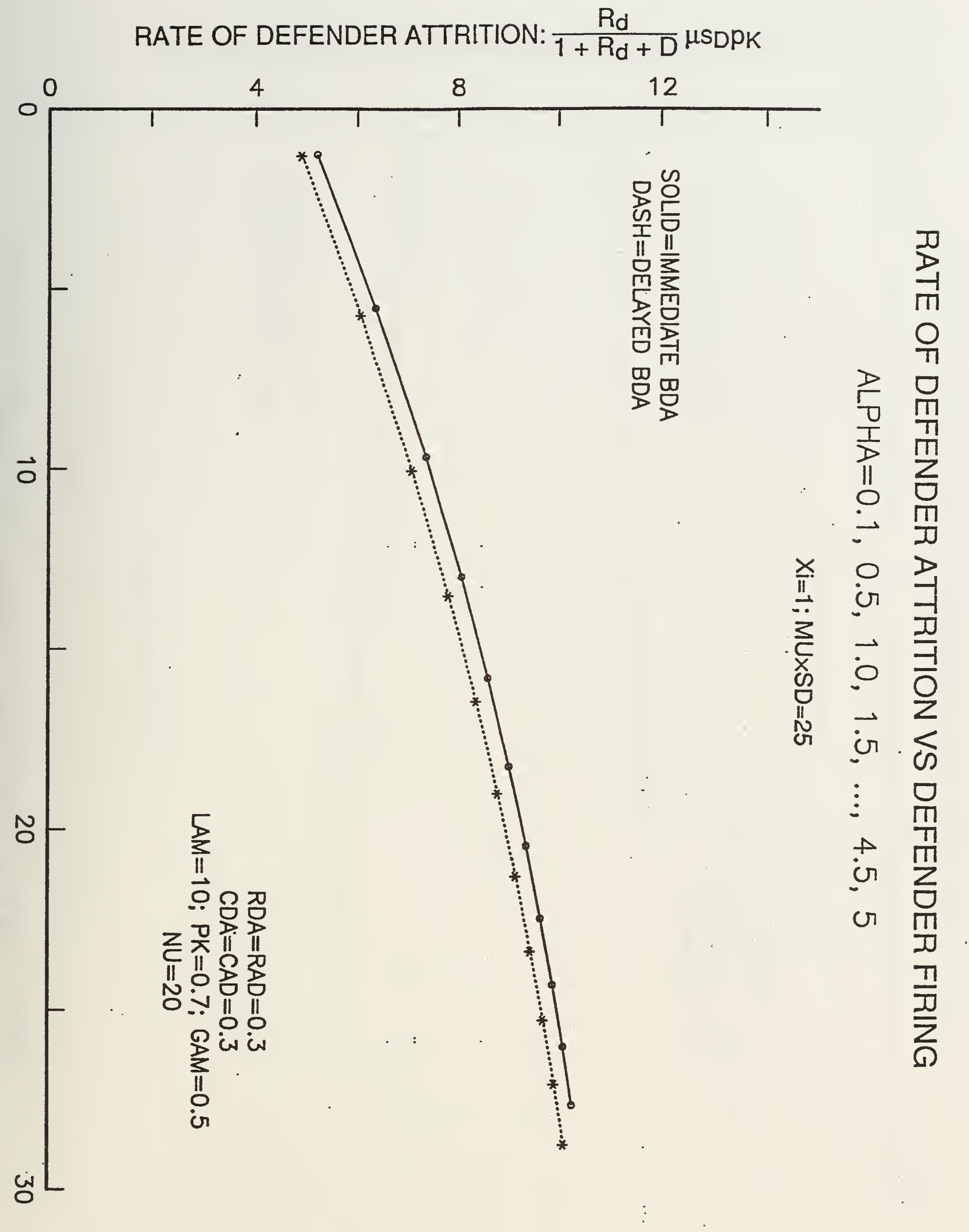

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=1; MUxSD=25
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30

Figure 3.1

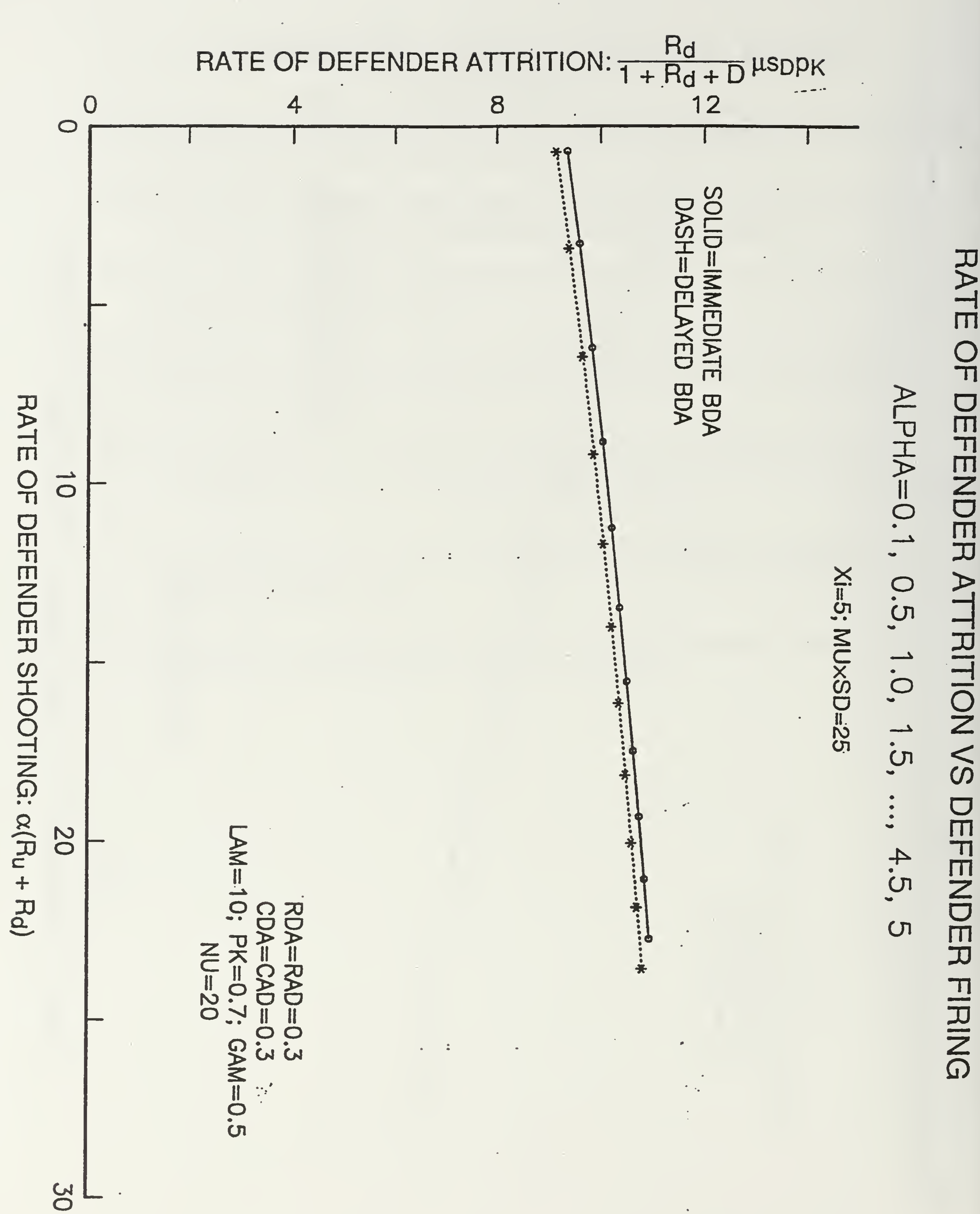

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=5; MU×SD=25
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.2

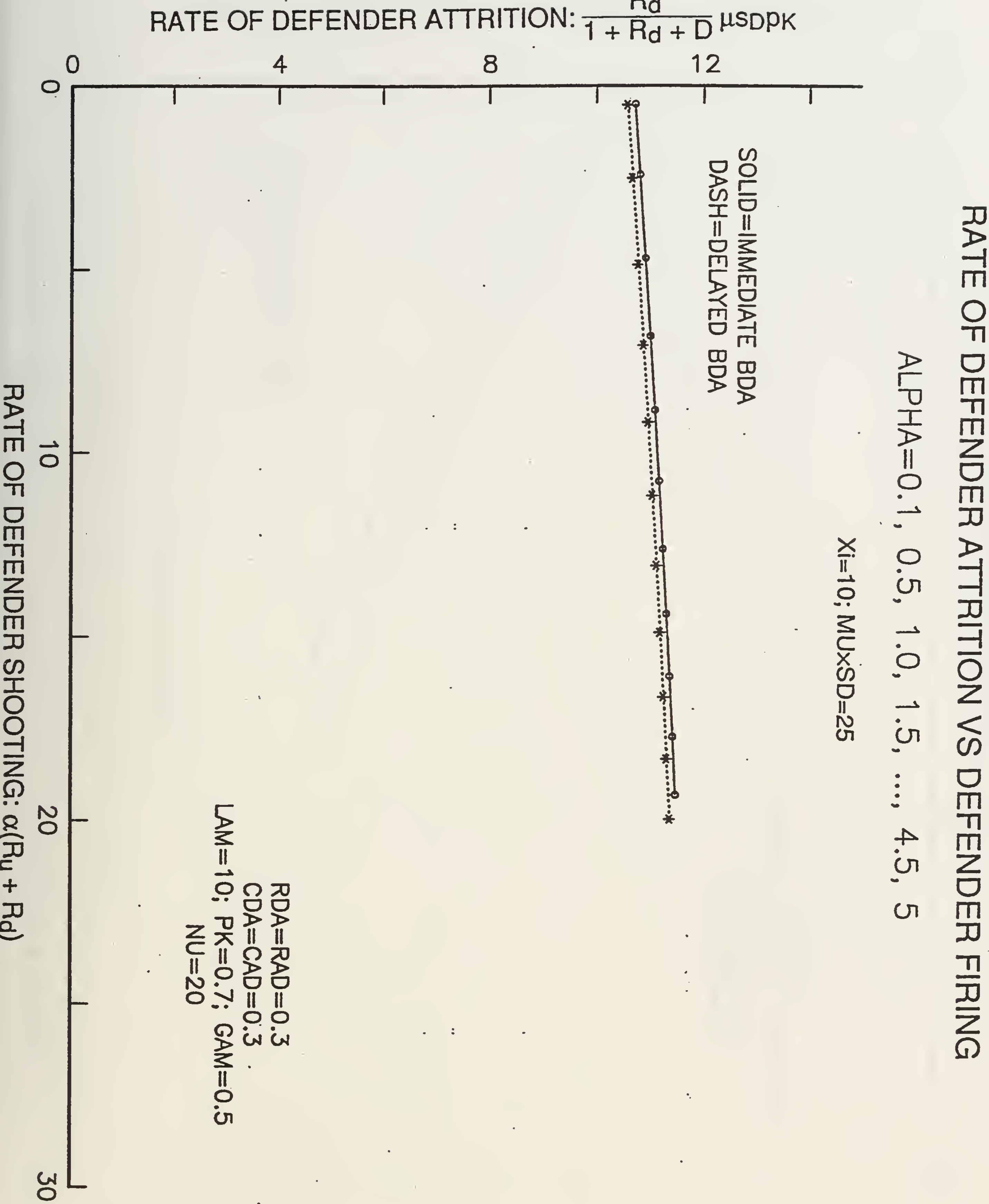

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=10; MUxSD=25
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu_{SD} p_K$
0
4
8
12
0
10
20
30
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.3

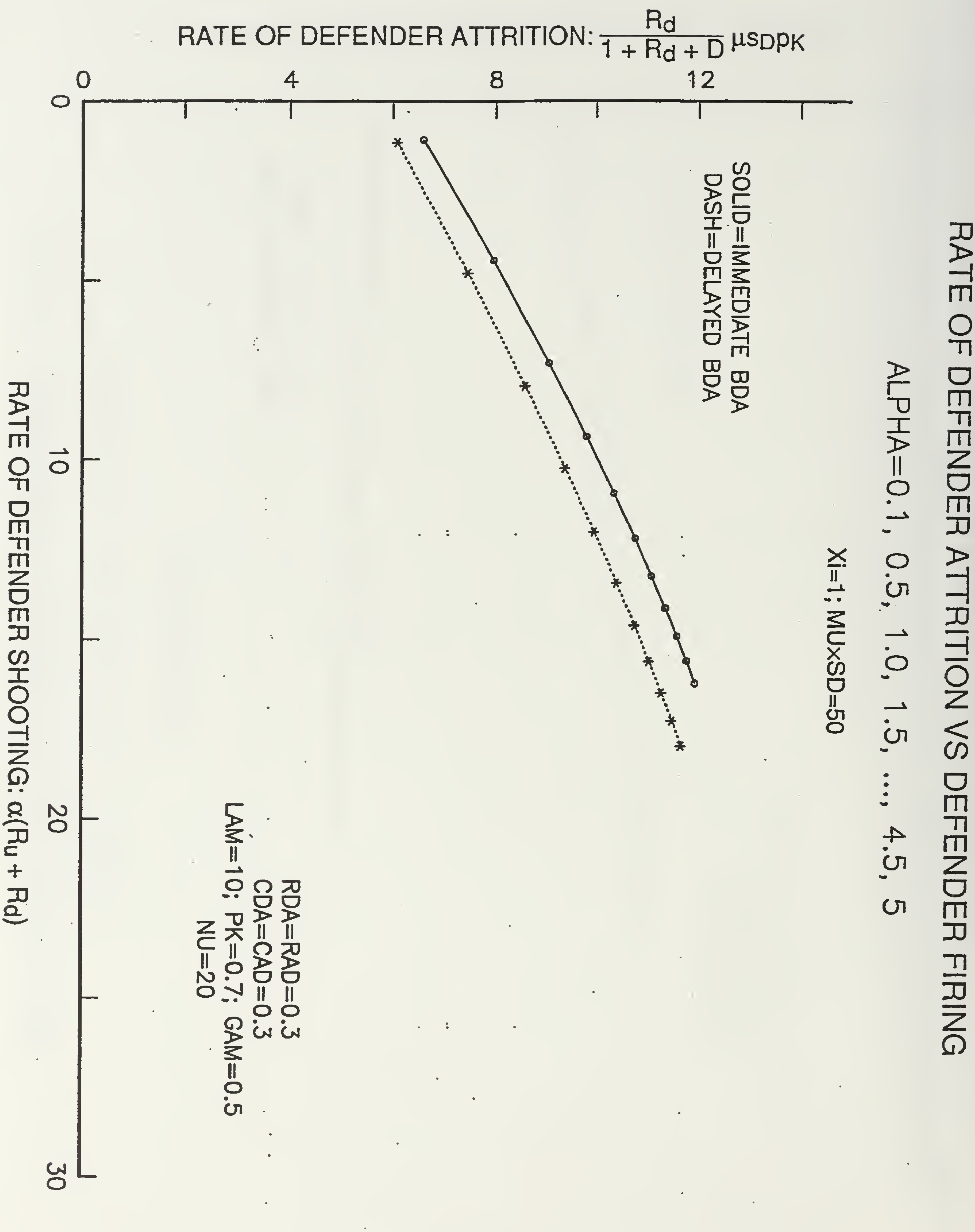
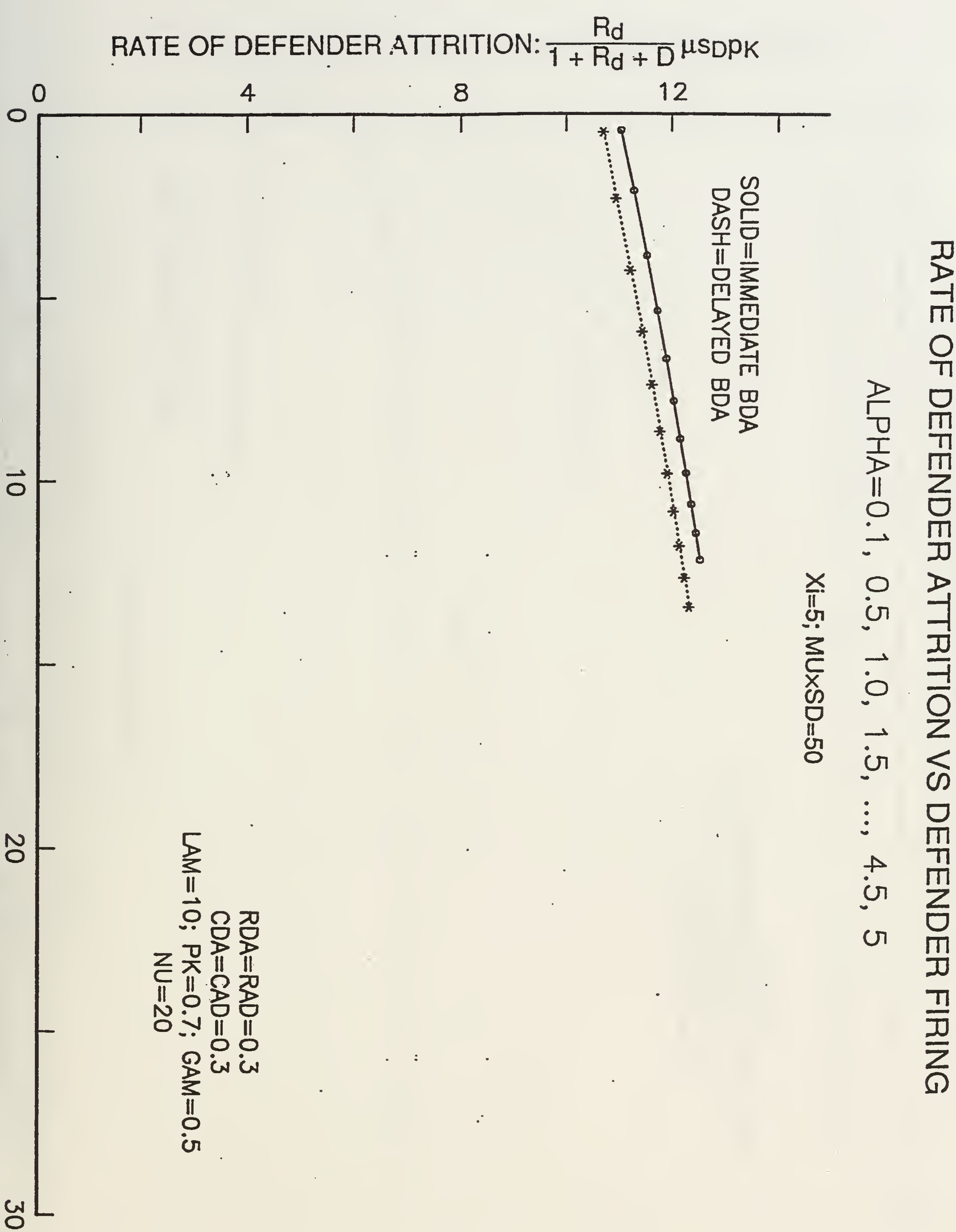
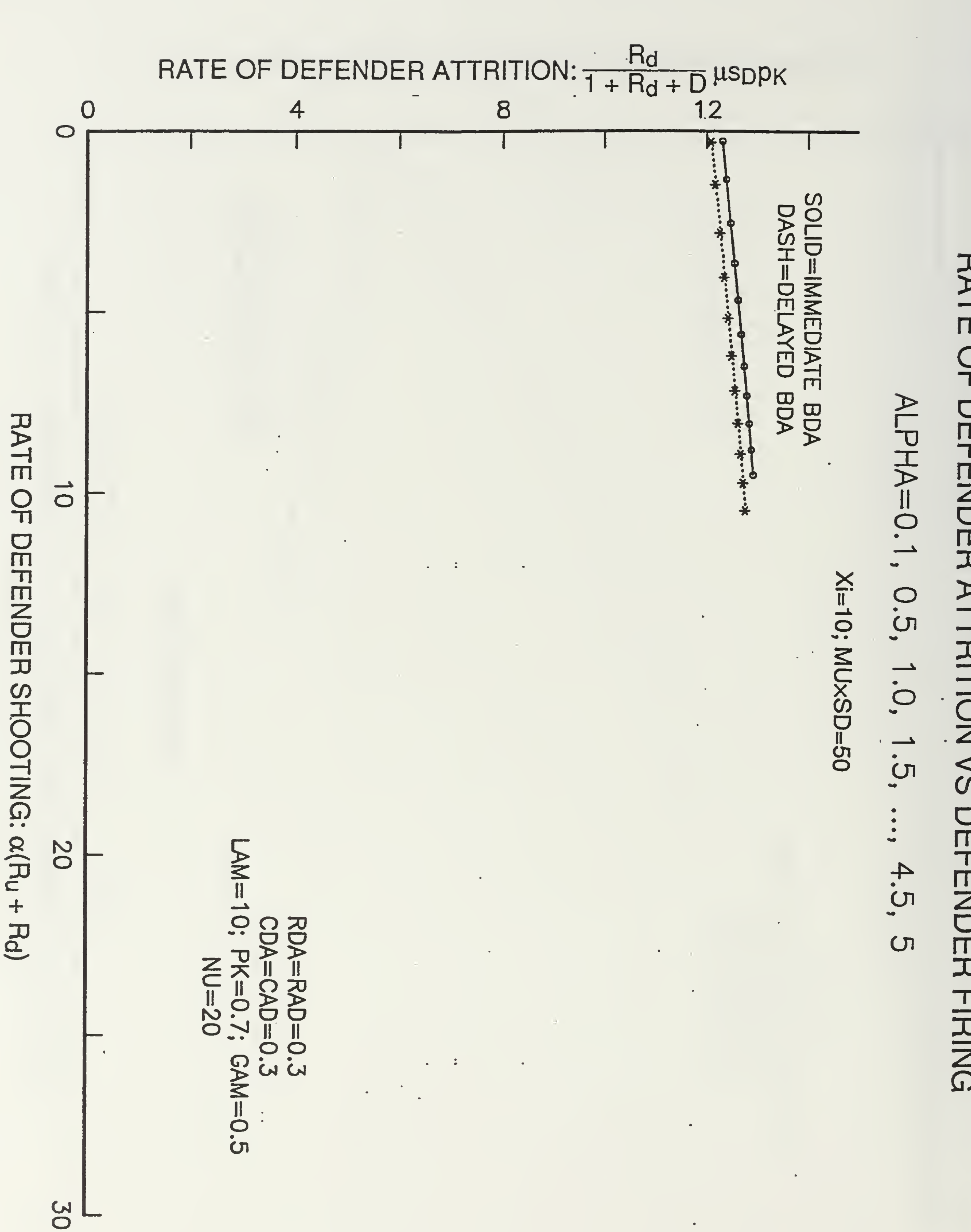

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=1; MUxSD=50
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.4



RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=5; MUxSD=50
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.5



RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
Xi=10; MUxSD=50
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=0.3
CDA=CAD=0.3
LAM=10; PK=0.7; GAM=0.5
NU=20
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D}\mu s_D p_K$
0
4
8
12
0
10
20
30
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.6

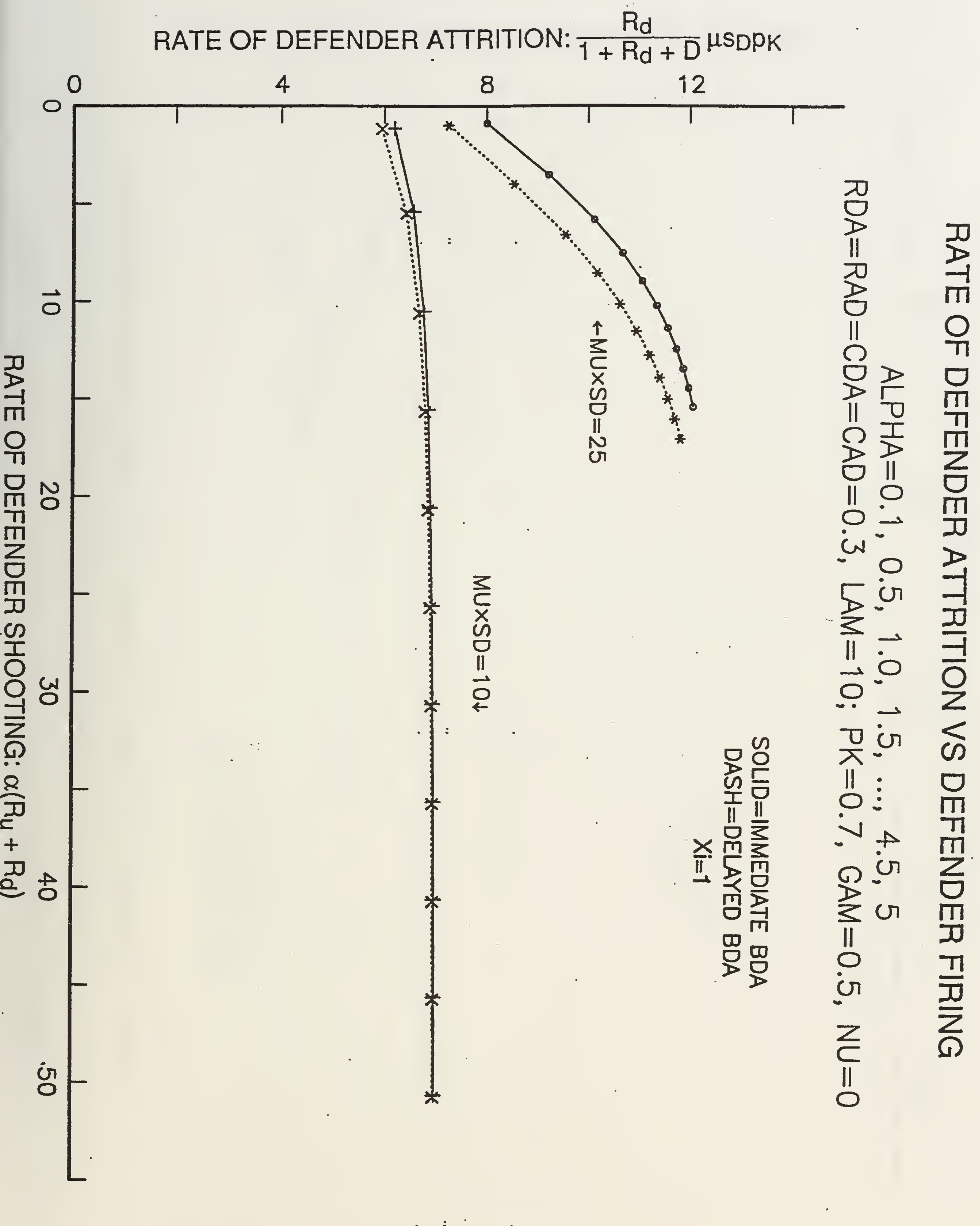

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
RDA=RAD=CDA=CAD=0.3, LAM=10; PK=0.7, GAM=0.5, NU=0
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
Xi=1
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D}\mu s_D p_K$
0
4
8
12
←MU×SD=25
MU×SD=10↓
0
10
20
30
40
50
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.7

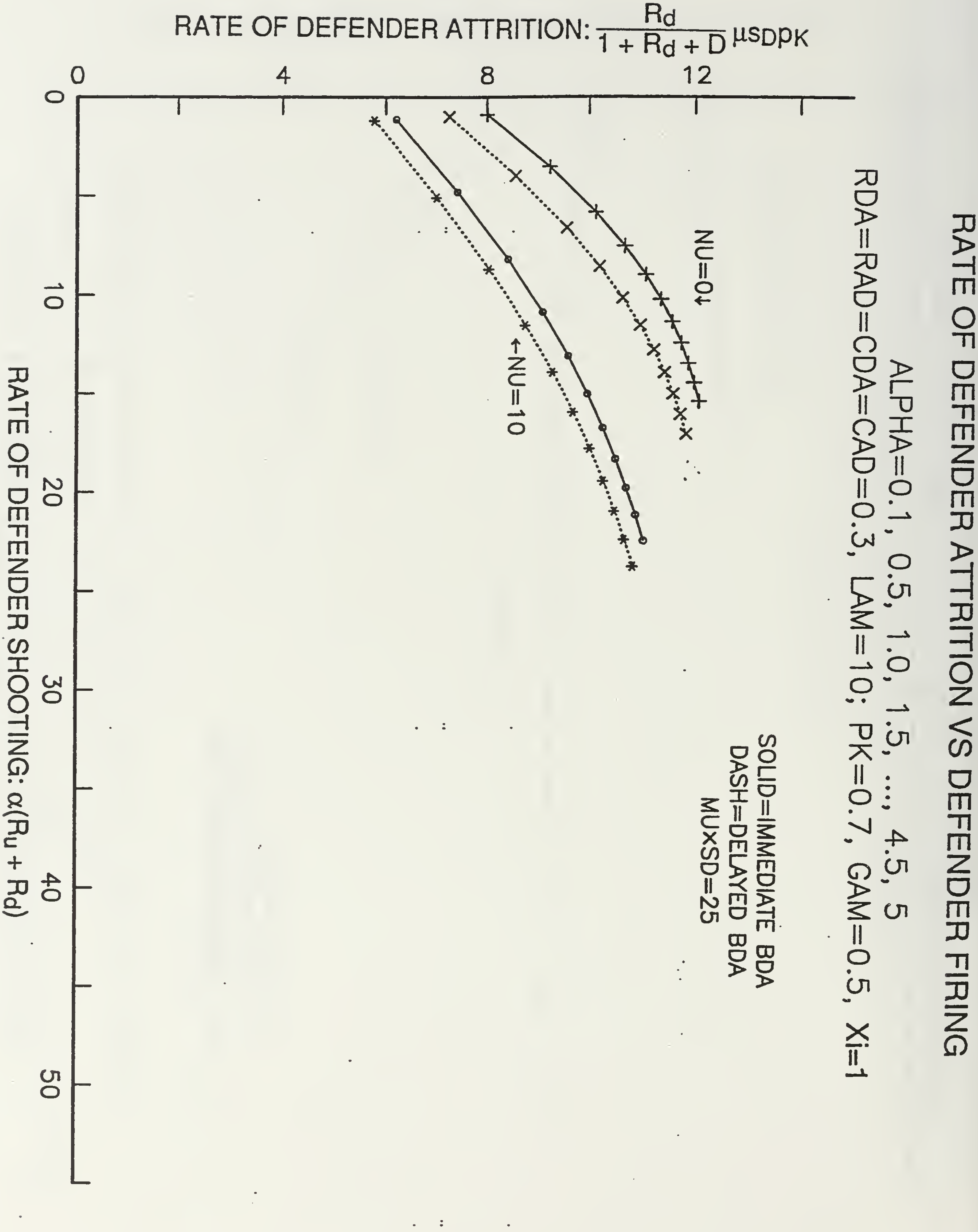

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
RDA=RAD=CDA=CAD=0.3, LAM=10; PK=0.7, GAM=0.5, Xi=1
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
MU×SD=25
NU=0
NU=10
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu_{SD} p_K$
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$
0
4
8
12
10
20
30
40
50

Figure 3.8

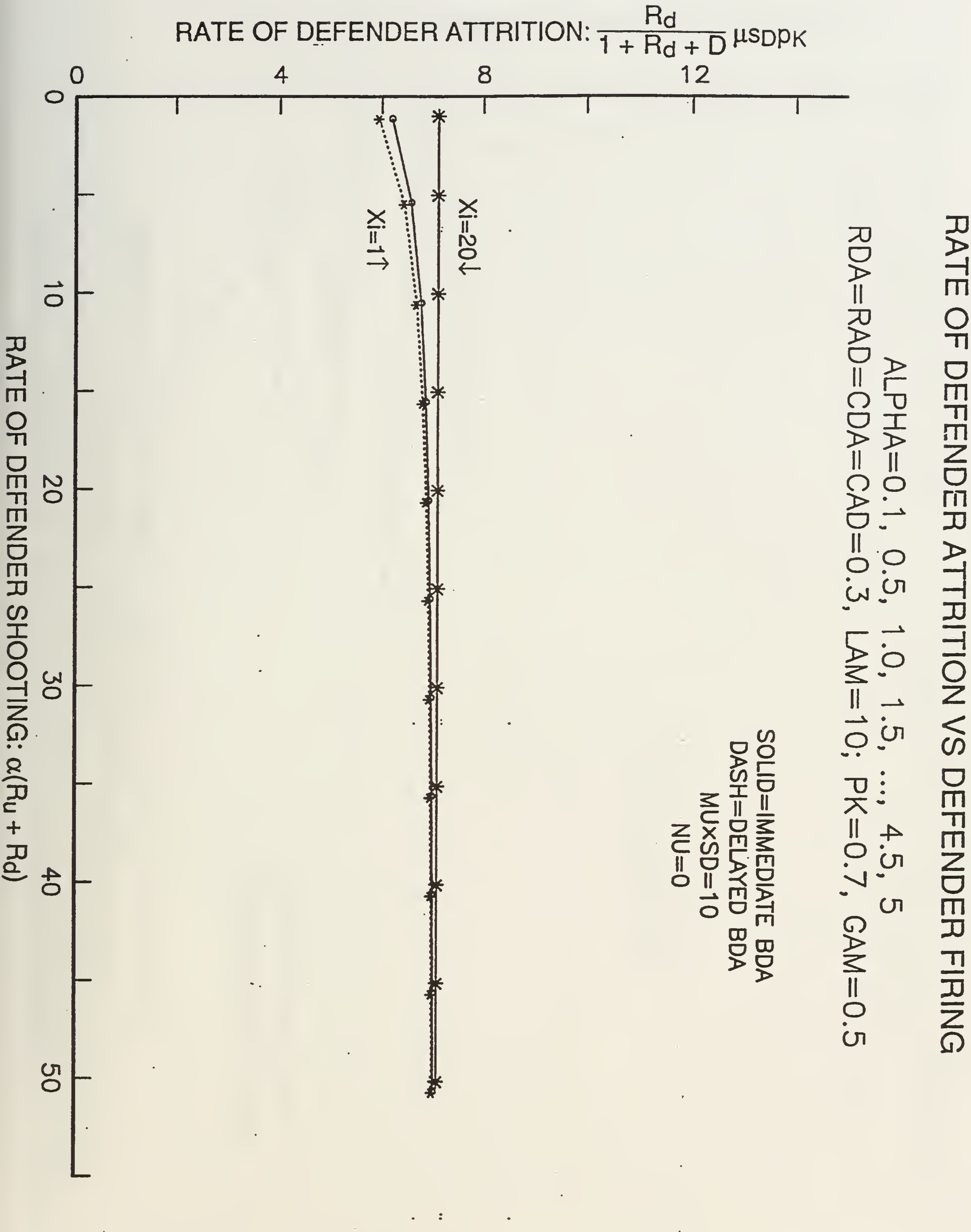
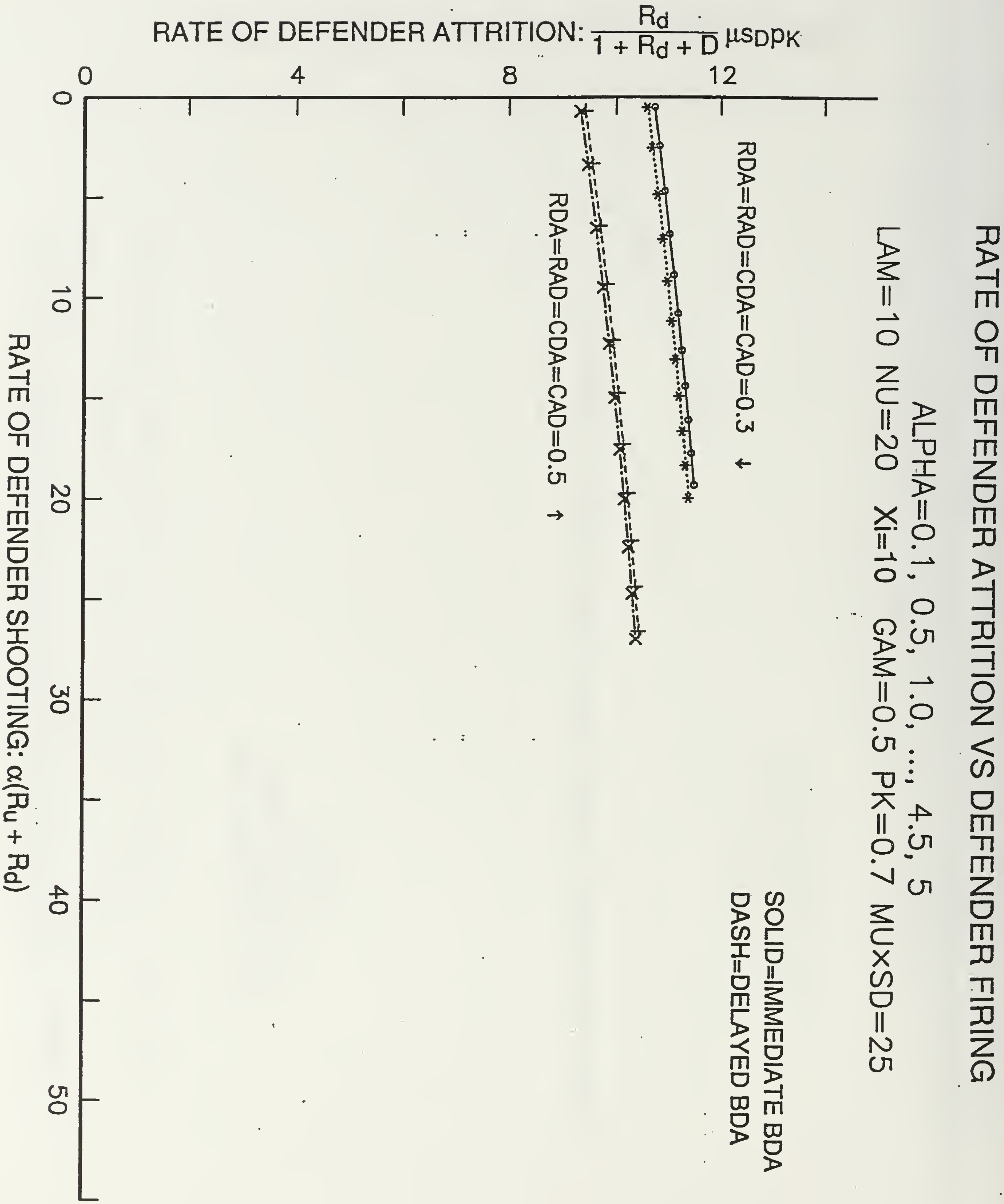

RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
ALPHA=0.1, 0.5, 1.0, 1.5, ..., 4.5, 5
RDA=RAD=CDA=CAD=0.3, LAM=10; PK=0.7, GAM=0.5
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
MU×SD=10
NU=0
Xi=20↓
Xi=1↑
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30
40
50
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.9


RATE OF DEFENDER ATTRITION VS DEFENDER FIRING
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LAM=10 NU=20 Xi=10 GAM=0.5 PK=0.7 MU×SD=25
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RDA=RAD=CDA=CAD=0.3
RDA=RAD=CDA=CAD=0.5
RATE OF DEFENDER ATTRITION: $\frac{R_d}{1 + R_d + D} \mu s_D p_K$
0
4
8
12
0
10
20
30
40
50
RATE OF DEFENDER SHOOTING: $\alpha(R_u + R_d)$

Figure 3.10

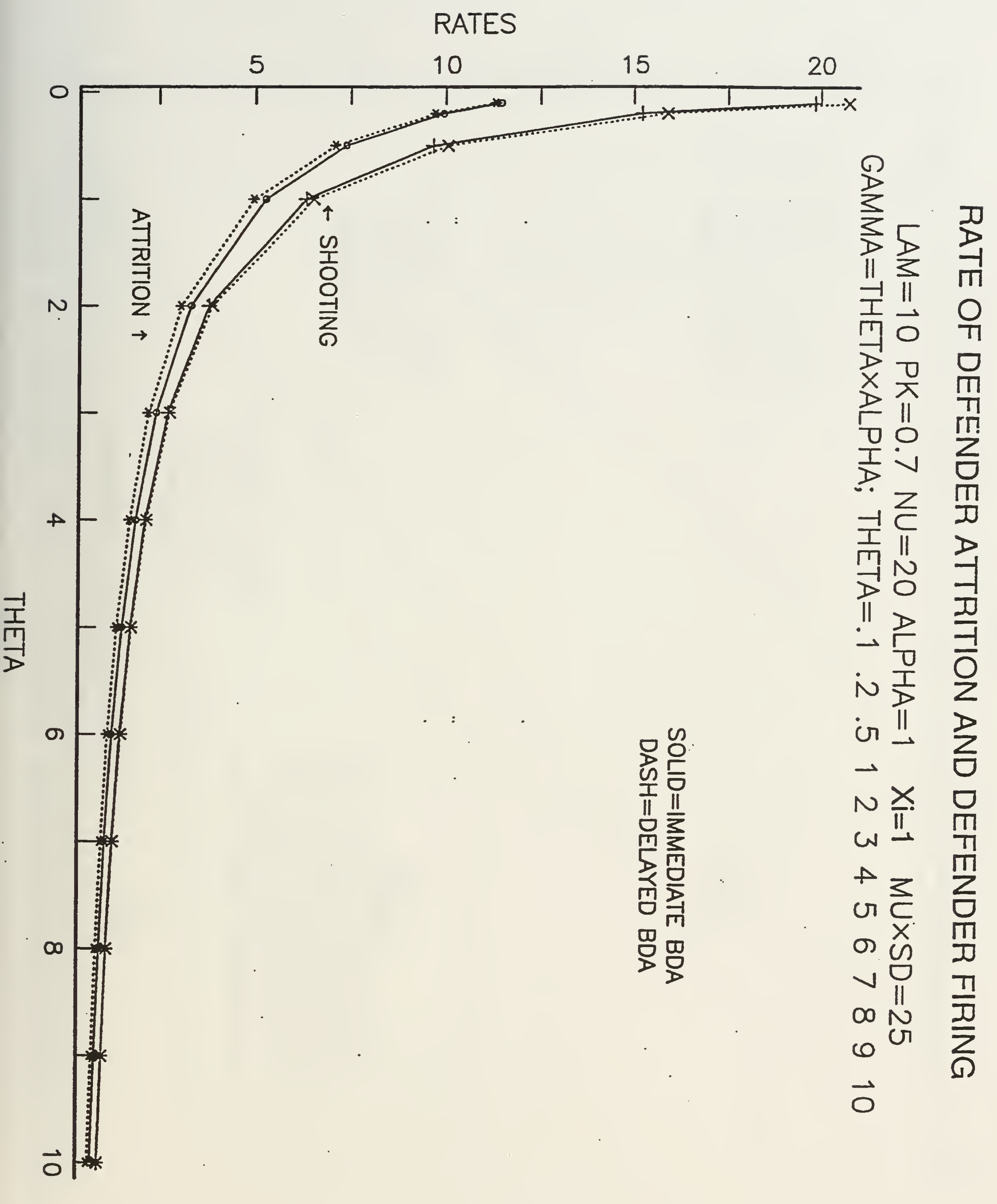

RATE OF DEFENDER ATTRITION AND DEFENDER FIRING
LAM=10 PK=0.7 NU=20 ALPHA=1 Xi=1 MU×SD=25
GAMMA=THETA×ALPHA; THETA=.1 .2 .5 1 2 3 4 5 6 7 8 9 10
SOLID=IMMEDIATE BDA
DASH=DELAYED BDA
RATES
5
10
15
20
0
2
4
6
8
10
THETA
← SHOOTING
ATTRITION →

Figure 3.11

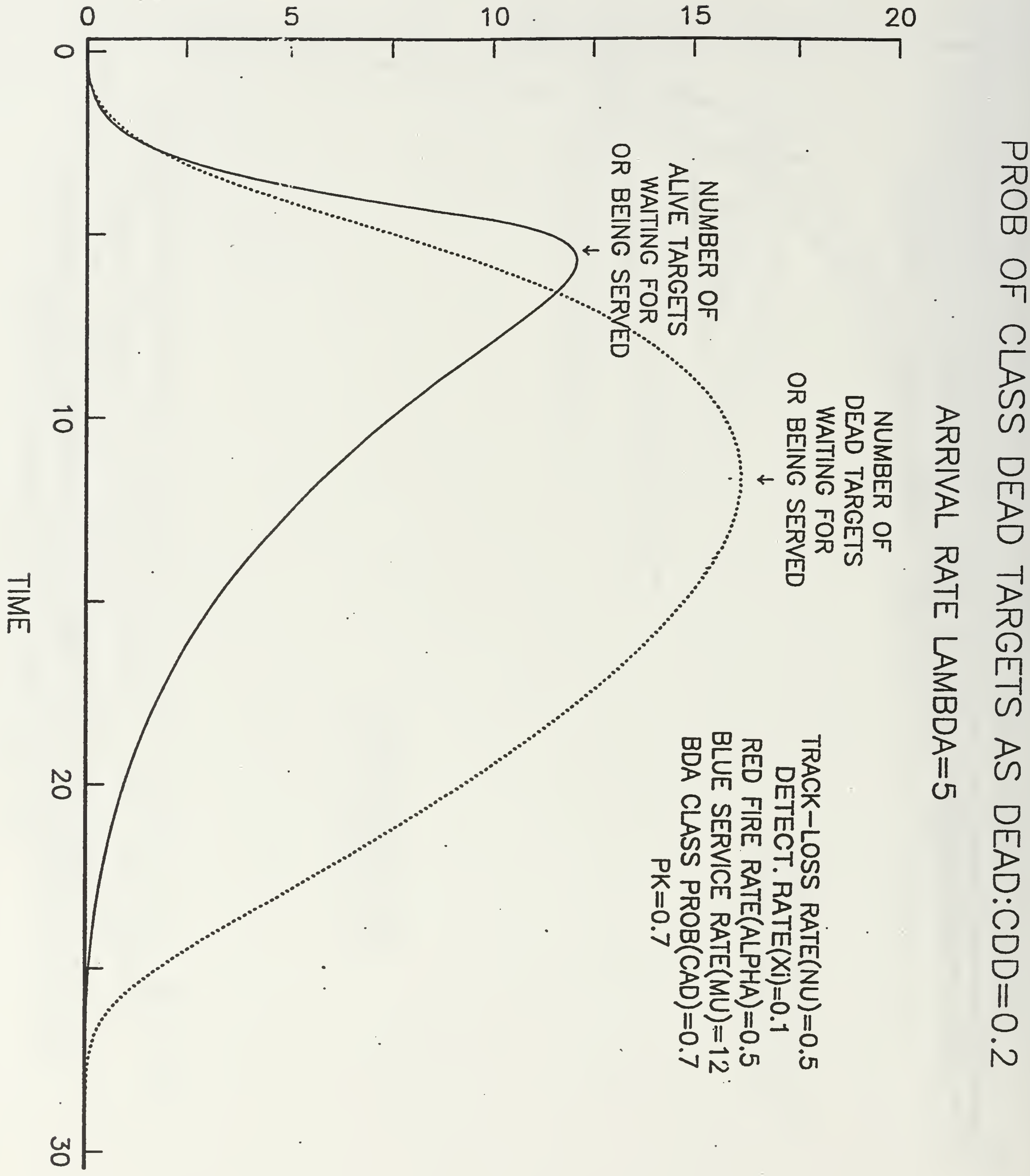
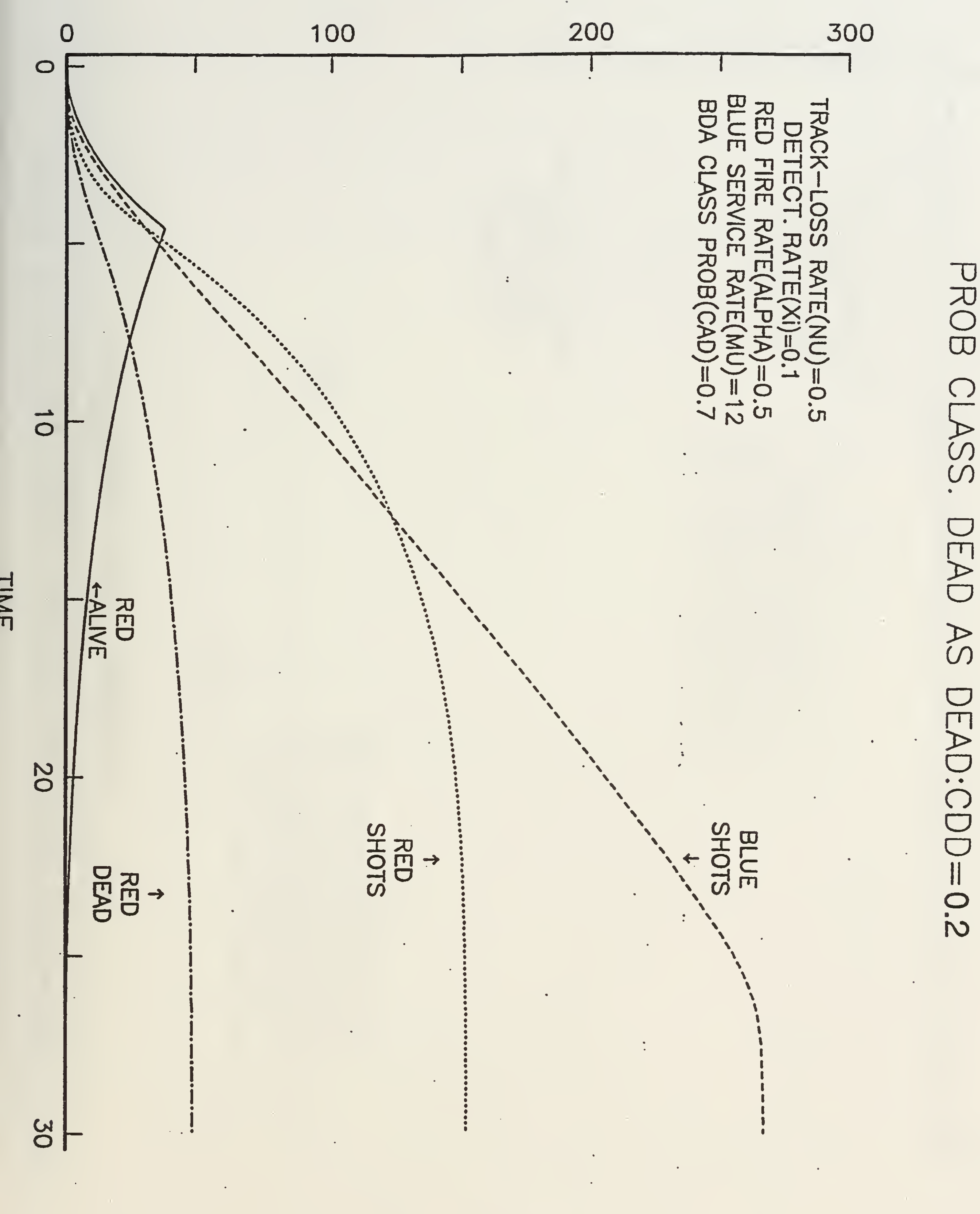
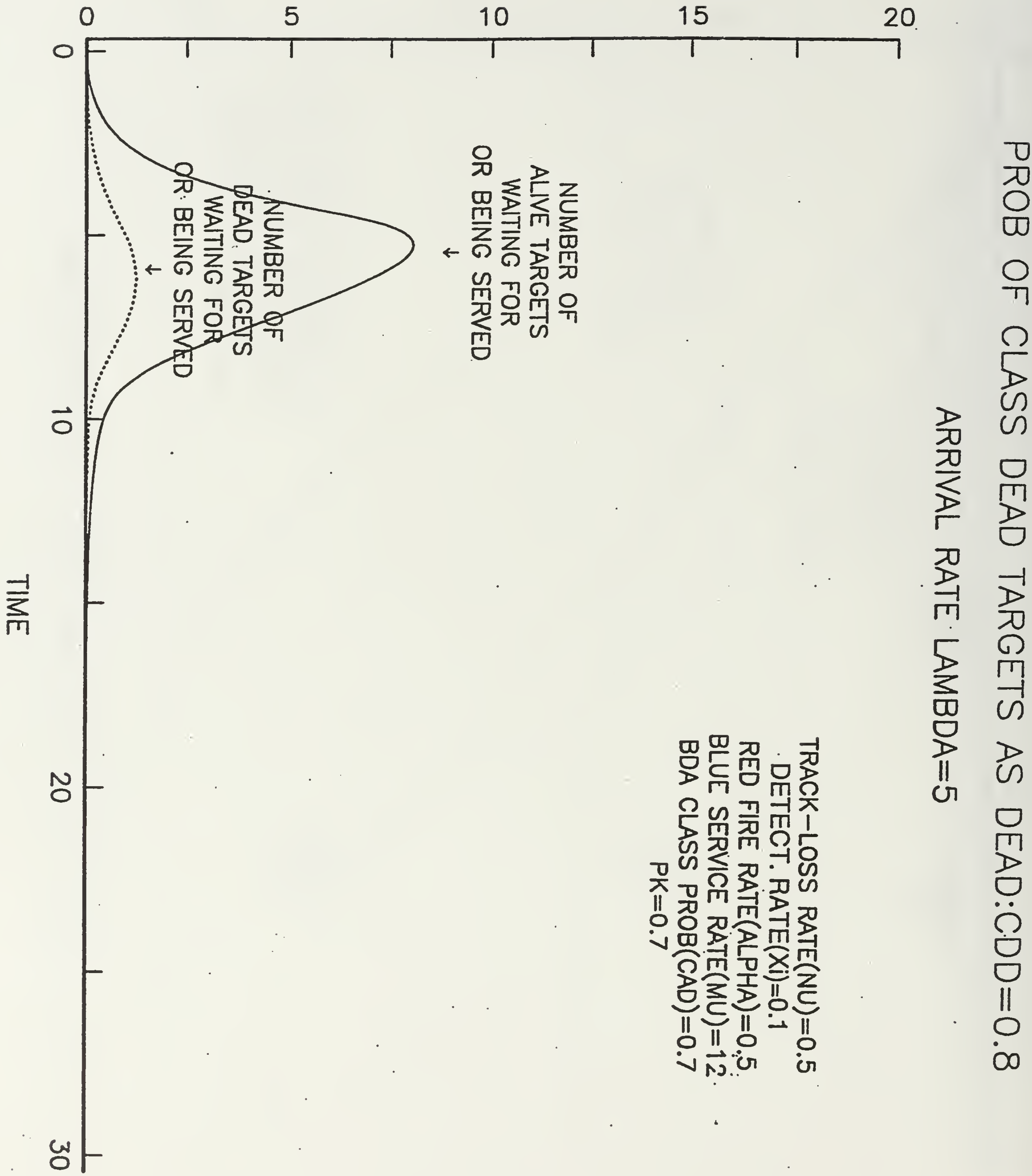

PROB OF CLASS DEAD TARGETS AS DEAD:CDD=0.2
ARRIVAL RATE LAMBDA=5
TRACK–LOSS RATE(NU)=0.5
DETECT. RATE(XI)=0.1
RED FIRE RATE(ALPHA)=0.5
BLUE SERVICE RATE(MU)=12
BDA CLASS PROB(CAD)=0.7
PK=0.7
NUMBER OF ALIVE TARGETS WAITING FOR OR BEING SERVED
NUMBER OF DEAD TARGETS WAITING FOR OR BEING SERVED
0
5
10
15
20
0
10
20
30
TIME

Figure 3.12



PROB CLASS. DEAD AS DEAD:CDD=0.2
TRACK—LOSS RATE(NU)=0.5
DETECT. RATE(Xi)=0.1
RED FIRE RATE(ALPHA)=0.5
BLUE SERVICE RATE(MU)=12
BDA CLASS PROB(CAD)=0.7
0
100
200
300
0
10
20
30
TIME
RED
←ALIVE
BLUE
SHOTS
RED
SHOTS
RED
DEAD

Figure 3.13


PROB OF CLASS DEAD TARGETS AS DEAD:CDD=0.8
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TRACK—LOSS RATE(NU)=0.5
DETECT. RATE(Xi)=0.1
RED FIRE RATE(ALPHA)=0.5
BLUE SERVICE RATE(MU)=12
BDA CLASS PROB(CAD)=0.7
PK=0.7
NUMBER OF ALIVE TARGETS WAITING FOR OR BEING SERVED
NUMBER OF DEAD TARGETS WAITING FOR OR BEING SERVED
TIME
0
5
10
15
20
0
10
20
30

Figure 3.14

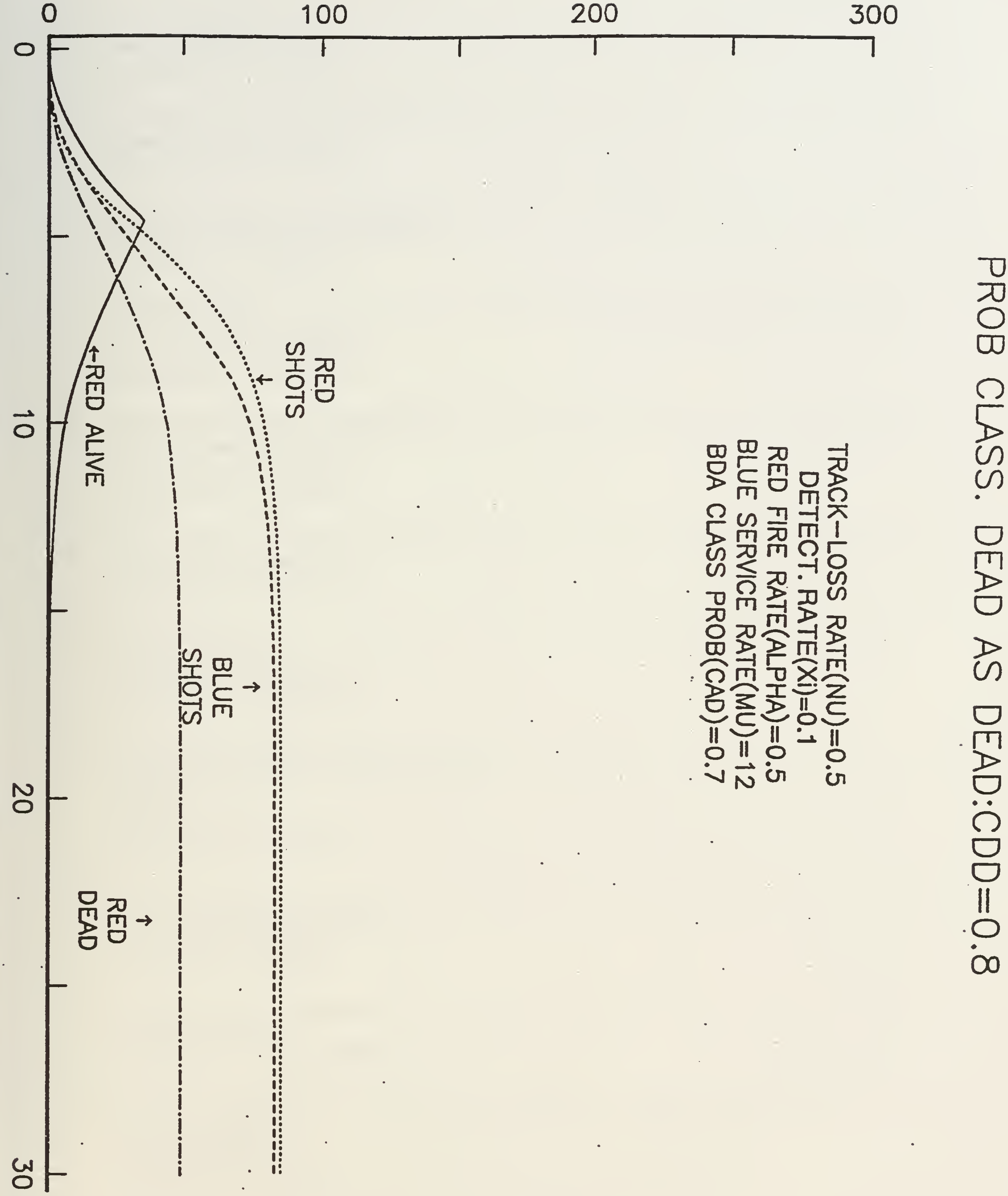
PROB CLASS. DEAD AS DEAD:CDD=0.8
TRACK—LOSS RATE(NU)=0.5
DETECT. RATE(Xi)=0.1
RED FIRE RATE(ALPHA)=0.5
BLUE SERVICE RATE(MU)=12
BDA CLASS PROB(CAD)=0.7
0
100
200
300
0
10
20
30
RED SHOTS
BLUE SHOTS
RED DEAD
←RED ALIVE
TIME

Figure 3.15

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